Please note the following:

1. Do not put your name on your submission.
2. Write only on the front of your answer pages.
3. Start your answer to each new question on a new sheet of paper.
4. In the top right corner of each page of your submission, include your chosen letter and the page number.

I advise you to read through the exam in its entirety before beginning the exam. This will help you properly allot your time.

Stay calm.

Best,

Tamah
1. A firm hires a consultant to improve its human resource management (HRM). The consultant can work hard on the improvement of HRM \((e = 2)\), and in this case there is considerable improvement for sure. If the consultant doesn't work hard \((e = 1)\), with probability \(p = 0.6\) the considerable improvement can still be achieved; otherwise the improvement is only slight. The cost of working is \(c(2) = 1\), and \(c(1) = 0\). The firm's payoff with considerably and slightly improved HRM are, respectively, \(Y_C = 10\) and \(Y_S = 5\). Whether the consultant worked hard is not observable, but the degree of improvement of HRM is contractible. The firm offers a contract to the consultant, specifying the fixed payment \(w\), paid always, independently of the degree of improvement, and bonus \(b\) paid in case of considerable improvement. Both the firm and the consultant are risk-neutral and have zero reservation utilities. There is limited liability constraint: the payment from the firm to the consultant should always be non-negative. The timing is standard.

(a) What is the optimal contract for the firm to offer the consultant?
(b) After accepting the contract, but before deciding on effort, the consultant learns whether there will be considerable improvement in case \(e = 1\), and only after this decides whether \(e = 2\) or \(e = 1\). What is the optimal contract for the firm to offer? Does the firm gain or lose from the consultant having extra information? Give an intuitive explanation.
2. Suppose two risk averse bidders participate in a first-price auction. Specifically, each bidder’s utility over a monetary outcome $x$ is given by $x^{\frac{3}{2}}$. Further, each player’s valuation for the good is uniformly distributed between 0 and 1.

(a) Suppose that bidder 2 bids $\frac{2}{3}v$ when her valuation is $v$. What is bidder 1’s expected utility from bidding $b$ when her valuation is $w$?

(b) Show that bidding $\frac{2}{3}v$ is an equilibrium.

(c) Compare this to the expected revenue from a second price auction in this situation. Give an intuitive explanation for this result.
3. Consider the following pure exchange economy with three agents. There are two goods, $x$ and $y$. All consumers have utility function:

$$u(x, y) = x^{\frac{1}{4}} y^{\frac{3}{4}}.$$ 

Agent 1 and 2 are endowed with 1 unit of $x$ and 2 units of $y$. Agent 3 is endowed with 2 units of $x$ and 1 unit of $y$. Solve for a Walrasian equilibrium of this exchange economy.
4) The Department of Justice (DOJ) is charged with enforcing federal antitrust laws with regard to mergers and acquisitions. A central issue in many of their reviews is the extent of the market, i.e., how many competitors/goods are there in a given market. This often hinges on whether the cross-price elasticities among a set of goods are non-zero. To answer this question, DOJ often estimates demand systems for potentially related goods and test whether cross price effects are non-zero. If so, the goods are deemed to be substitutes/complements and the competitive implications of the merger for the related goods and their prices must be determined.

To answer the extent of the market question, DOJ typically estimate a demand system such as the following:

\[ s_i = \alpha_i + \sum_{i=1}^{N} \beta_{ni} \ln p_i + \lambda_i y \]

\[ \vdots \]

\[ s_N = \alpha_N + \sum_{i=1}^{N} \beta_{ni} \ln p_i + \lambda_N y \]

where \( s_i = p_i x_i / y \) is the expenditure share associated with good \( i \), \( p_i \) is the price of the \( i \)th good, \( y \) is income and \( (\alpha, \beta, \lambda) \) are estimable parameters. Assume that all prices and income are normalized by the price of the outside good, which has a strictly positive demand. In general, the parameter space equals \( 2N + N^2 \), and thus estimating all parameters can raise computational difficulties with moderate to large \( N \). How can economic theory be used to reduce the estimable parameter space in the context of the above demand system? Please state in general terms all restrictions implied by economic theory and their implications for the parameters in the above specification.
5) A consumer has an expected utility function given by \( u(w) = \ln(w) \) where \( w \) is wealth. He is offered the opportunity to bet on the flip of a coin that has a probability \( \pi \) of coming up heads. If he bets $x, he will have \( w+x \) if heads comes up and \( w-x \) if tails comes up. Solve for the optimal \( x \) as a function of \( \pi \). What is his optimal choice of \( x \) when \( \pi = 1/2 \)?
6) Consider a market with \( n \) identical firms producing a homogenous good. Each firm generates pollution emissions during production. Assume:

- \( P(\bar{q}) \): Inverse aggregate demand
- \( n \): number of firms
- \( q \): firm output
- \( C(q) \): cost per firm as a function of output
- \( e(q,n) \): emissions per firm as a function of \( q \) and \( n \)
- \( D(n,e(q,n)) \): social damage from \( n \) firms, each of which emits \( e(q,n) \) units of emissions. These damages are external to firms.

Show that in the long run (and in contrast to the Pigouvian tradition), the social optimum is not in general achieved by a tax on emissions if the industry is perfectly competitive. Are there special conditions under which the emissions tax is socially efficient?
**Instructions.** Do all 4 questions. Point values are shown in parentheses. You have 4 hours to complete the exam. If you know part but not all of the answer, put down what you know. A blank answer is guaranteed to earn no points, so even if you don’t know the complete answer, be sure to tell us what you do know. However, do not guess if you don’t know. Irrelevant statements will not hurt your score, but wrong statements will cost you points and so will be worse than saying nothing.

1. (30 points) Consider an economy where households maximize the following utility function:

   \[ U = E_t \sum_{s=t}^{\infty} \beta^s \ln c_t + \theta_m \ln \frac{M_{t+1}}{P_t} + \theta_a \ln (1 - N_t) \]

   Households supply labor and rent capital to firms. Households receive real wages \( w_t \) for each unit of labor provided and capital income \( r_t \) for each unit of \( K_t \). The households’ budget constraint is

   \[ c_t + \frac{M_{t+1}}{P_t} + K_{t+1} = w_t N_t + r_t K_t + \frac{M_t + T_t}{P_t} \]

   \( T_t \) represents lump-sum transfers from the government. Let \( \pi_{t+1} = P_{t+1}/P_t \) be the inflation rate. Firms are perfectly competitive and rent labor and capital from households to produce a homogeneous good according to a Cobb Douglas production function \( z_t K^\alpha_t L^1 - \alpha \). Productivity \( z_t \) is exogenous and follows an AR(1) process subject to i.i.d productivity shocks:

   \[ \ln z_t = \rho \ln z_{t-1} + \varepsilon_t \quad (\rho < 1). \]

   Steady state productivity \( z \) is equal to 1. Assume that the quantity of money supplied \( M_t^e \) evolves according to an exogenous process.

   a) (3 points) Explain briefly why in a competitive equilibrium wages and capital rental rates should equal the marginal productivity of labor and capital.

   b) (3 points) Write down the government’s budget constraint.

   c) (6 points) Derive the first order conditions for the household and for the firm.

   d) (9 points) Log-linearize the first order conditions and all relevant constraint(s).

   (NOTE: you do not need to find relevant steady state ratios such as \( \frac{\delta^e}{\delta^c}, \frac{\delta^c}{\delta^c} \) as a function of the parameters.) Show that, in equilibrium, hours worked are independent of the shocks, that output and hours are uncorrelated, and that real wages are perfectly correlated with output.

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e) (3 points) Show that monetary disturbances are neutral. Explain whether they are also superneutral.

f) (3 points) Assume now that the government consumes $G_t$ units of output each period and that $G_t$ is stochastic. What is the effect of government expenditure shocks on the correlation between real wages and output?

g) (3 points) Suppose there are labor contracts where the nominal wage rate in period $t$ is fixed before shocks in period $t$ are realized. Explain why a monetary disturbance will produce a contemporaneous negative correlation between real wages and output.

2. (20 points) Consider a simple forward-looking model of the form

$$x_t = E_t x_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}) + u_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t$$

where $x_t$ is the output gap, $\pi_t$ is the inflation rate, $i_t$ is the risk free nominal interest rate, and $u_t$ and $e_t$ represent i.i.d. mean zero shocks in the economy. Suppose policy reacts to the output gap:

$$i_t = \delta x_t$$

a) (4 points) Explain what an “output gap” means in the context of this model.

b) (8 points) Find values of $\delta$ that ensure a unique stationary equilibrium.

c) (8 points) Assume $\delta$ takes a value that ensures determinacy. Solve the model and explain the responses of $x_t$, $\pi_t$, and $i_t$ to deviations in $u_t$.

3. (30 points) Consider the following economy. The production function is given by:

$$Y(t) = F[K(t), L(t)]$$

where $F$ has the neoclassical properties (positive and decreasing marginal returns). A fraction $\delta$ of capital depreciates at each point in time. The rate of population growth is $n > 0$. Households have the following utility function:

$$\max \int_0^\infty c(t)^{1-\theta} - 1 \frac{1}{1-\theta} e^{-\rho t} e^{nt} dt$$

where $c(t)$ is consumption per capita. Households receive a lump-sum transfer from the government equal to $\phi$ at each point in time. Households provide labor $L(t)$ and rent out capital $K(t)$ to perfectly competitive firms. Households bear the cost of depreciation.

a) (4 points) State the optimal growth problem. Be sure to include all of the constraints.
b) (4 points) State the Hamiltonian function for this problem and the corresponding first-order conditions. Also state the transversality condition.

c) (4 points) Using your answers above, derive a pair of differential equations for the variables \((k, c)\). Please show your derivations.

d) (5 points) What is the economic interpretation of the multiplier in the Hamiltonian function? [Give a short intuitive answer.]

e) (8 points) Assume the baseline economy starts in steady state. The government unexpectedly decides to set permanently \(\phi = 0\). Draw the phase diagram for both the baseline and the modified economy, indicating what is different. Be sure to label your diagram clearly. How does the equilibrium change? What happens on impact to \(c\) and \(k\)?

f) (5 points) How does \(\phi\) affect the level of steady state consumption? Why? [Give an intuitive explanation.]

4) (20 points) Consider an economy where the saving rate is exogenous. The production function takes a Cobb-Douglas form, \(Y(t) = K(t)^{0.5}L(t)^{0.5}\). Capital depreciates at a constant per-period rate \(\delta = 0.1\). The saving rate \(s\) equals 0.2.

a) (3 points) What are the steady state values of \(k = \frac{K}{L}\), \(y = \frac{Y}{L}\), and \(i = \frac{i}{L}\)?

b) (3 points) Find the values for \(k\) and \(y\) if the economy operates at the "Golden Rule" level of capital accumulation.

c) (6 points) Find the saving rate required to achieve the golden rule allocation. Is the initial saving rate higher, equal, or lower than \(s^{old}\), i.e. the one implied by the "Golden Rule" allocation?

d) (8 points) Starting from the initial saving rate, assume that the saving rate becomes \(s^{old}\). What would be the immediate and long-run effects on \(c, k\), and \(y\)? Draw the path of these variables. Please report them on a separate graph (one for each variable), with time on the horizontal axis and the values of \(c, k\) and \(y\) on the vertical axis. Is the economy dynamically inefficient?
Proposition 1. (from Woodford, 1999). Consider a linear rational-expectations model of the form

\[ E_t z_{t+1} = A z_t + e_t \]

where \( z_t \) is a 2-vector of nonpredetermined endogenous state variables, \( e_t \) is a vector of exogenous disturbance terms, and \( A \) is a 2 \( \times \) 2 matrix of coefficients. Rational-expectations equilibrium is determinate if and only if the matrix \( A \) has both eigenvalues outside the unit circle (i.e., with modulus \(|\lambda| > 1\)). This condition in turn is satisfied if and only if either (Case 1)

\[ \det A > 1 \]

\[ \det A - \tr A > -1 \]

\[ \det A + \tr A > -1 \]

or (Case 2)

\[ \det A - \tr A < -1 \]

\[ \det A + \tr A < -1 \]