(1) For much of the 1990s the Japanese economy has been characterized as being in a recession with output less than would have been expected on the basis of previous trends. Japanese economic policy has focused on debt-financed public works projects as a remedy. The failure of these projects to solve the problem has led to recommendations for tax cuts as an additional remedy. The Japanese central bank believes monetary policy is already very expansionary because the nominal interest rate in Japan is below one percent.

The policy makers who have chosen these policies must have a hypothesis about the cause of the shortfall in Japanese output. Based on the description in the preceding paragraph explain what that hypothesis must be and evaluate the argument that tax cuts will help to end the recession and monetary policy will not.

(2) Consider a Sidrauski (or Money-in-the-Utility-Function) model and two alternative processes for the evolution of the nominal money supply: (a) \( \mu_t = k + e_t \) and (b) \( \mu_t = m + \theta \mu_{t-1} + e_t \), where \( \mu_t \) is the growth rate of the nominal money supply, \( e_t \) is a serially independent random variable with zero mean and finite variance, \( k \) and \( m \) are arbitrary constants and \( 0 < \theta < 1 \).

Explain why a non-zero realization of \( e_t \) will affect the short-run equilibrium value of \( (M/P)_t \) for one money growth rule and not for the other.

(3) "The key to understanding the sources of business cycles is the behavior of the real wage rate." Discuss this statement with respect to alternative theories of the business cycle. What role does labor supply play in these theories?

(4) It has been argued that dynamic inconsistency causes central banks to have an inflation bias, creating more inflation than is desirable. Specify a model and use it to explain carefully the argument for this view. Does this mean that a rule for monetary policy is preferable? Discuss some alternative solutions to this problem. Include in your answer any criticisms you have of this approach to analyzing policy.

(5) In the typical textbook model of investment, the optimal capital stock is determined by the schedule for the marginal product of capital and the user-cost of capital. Investment occurs to equate the capital stock with the optimal capital stock. This implies that investment rates approach infinity whenever the optimal capital stock increases which is clearly unrealistic. How can the textbook model be modified to eliminate this unrealistic behavior? Does such a modification imply stable investment expenditures? Should we expect a symmetric response of investment to changes in the user-cost of capital? Include in your answer a definition of the user-cost of capital.
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1. **Producer Theory**

For the production function $y = x_1^{1/2}x_2^{1/2}$, derive each of the following, where $y$ is output, $x$ is the vector of inputs and $w$ is the vector of input prices:

(a) conditional factor demand: $x_i^*(y,w)$
(b) cost function $C(y,w)$
(c) technical rate of substitution between factors $i$ and $j$: $\tilde{c}x_i^* : \tilde{c}x_j^*$
(d) profit function $\Pi^*(p,w)$

2. **Several short questions**

(a) Consider the following game represented in its extensive form, where the first number in each pair represents the payoff of player 1 and the second number that of player 2

```
Player 1
    a
   /  \
(0,2) /    \
   /      \\A
(1,-2)   B
```

Player 2

Identify both of the Nash equilibria of this game. Are either of them subgame perfect equilibria? What do you think will be the outcome of the game and why?

(b) Suppose that the pizza industry is competitive and that the unit cost of making a pizza is $5. Also suppose that David values one pizza at $8 wishes to consumer no more than one. Wally values pizza at $6 and wishes to consume no more than one. If the government levies a $2 tax on pizza, how great is the burden of the tax on David and on Wally, and how great is the deadweight loss due to the tax?

(c) A criminal presumably considers both the fine he must pay if caught and the probability of being caught. If the expected fine is held constant, does it deter crime to raise the fine paid if caught, or does it encourage crime?

(d) Wage rates are high in New York City compared to the rest of the United States; so too are living costs. Does this mean that the marginal product of labor is greater in New York City than it is in the rest of the United States?
3. **POW Camp**

All prisoners in a wartime prisoner-of-war camp receive identical weekly allotments from the Red Cross that include \( X = 2 \) packs of cigarettes and \( Y = 4 \) bars of chocolate. All of the prisoners have identical utility functions: \( U = X^{1/2}Y^{1/2} \).

(a) What is the price of cigarettes in terms of chocolate in the camp?

(b) Now suppose that a new set of prisoners is brought to the camp, and that each of these also receives the same weekly allotments of \( X = 2 \) packs of cigarettes and \( Y = 4 \) bars of chocolate, but their preferences differ from those of the original prisoners. Trade ensues and as a result the price of cigarettes in terms of chocolate changes. Are the original prisoners now better off than before, worse off than before, or does the answer depend upon whether the price of cigarettes in terms of chocolate rose or fell?

4. **Monopsony, regional integration and factor prices**

The country of Fractotia is composed of 1,000 identical regions. Each region has a single firm that produces output, \( y \), according to the same production function: \( y = 2x \), where \( x \) is the quantity employed of the one factor of production. Each region has an identical supply of the factor, given by \( x = w/3 \), where \( w \) is the factor’s wage. Because there is a national market for output, the firms are price-takers with respect to output price, \( p \). But there is no factor trade between regions and so each firm acts as a monopsonist in its own \( x \) market.

(a) Suppose that equilibrium output price initially is \( p = S3 \). What quantity is supplied by the industry? What is the factor price in each region? Suppose next that output price rises to \( S6 \). What now is the quantity supplied by the industry and the factor price in each region?

(b) Derive an expression for the industry supply curve and calculate the change in producer surplus for the industry due to the price increase from \( S3 \) to \( S6 \). Whose welfare is measured?

(c) Now suppose that factor trade opens up among the regions of Fractotia, forcing the 1,000 firms to act as price takers in the national \( X \) market. Under these new conditions, what would be the quantity supplied by the industry if output price were \( p = S3 \)? What would be the national factor price? What would be the quantity supplied and factor price if output price increased to \( S6 \).

(d) Under the new conditions, calculate the change in producer surplus for the industry due to the price increase from \( p = S3 \) to \( p = S6 \). Whose welfare is measured?
5. Production with uncertainty

A price-taking farmer produces crops, denoted y, using one variable input, pesticide, denoted x. Denote output price as p and input price as w. Output is random, with y equaling y_i with probability q_i and equaling 0 with probability 1-q_i. The effect of insecticide is to increase the probability of the high level of output:

\[ q = g(x), \quad g'(x) > 0. \]

Characterize the optimal choice of input for a risk-neutral farmer. What restriction or restrictions need to be placed on the function g(x) to ensure an interior solution (0 < x* < ∞). Using those restrictions, sign the comparative static effect on x from an increase in p.

6. Cartels and residual markets

The paper doll industry in the country of Cuttenclip operates as a cartel. The domestic demand curve that the cartel faces is given by Q = 100 - 10 P. The cartel’s costs of production are given by TC = Q + (1/80)Q^2.

The cartel has convinced the Cuttenclip government to ban imports of paper dolls into the country. While imports are banned, exports from the country are not, and the cartel can sell on the world market at the fixed price of $4. If the cartel exports, it must pay a one-time export license fee of $50.

(a) How much will the cartel sell to the domestic market and at what price? Will the cartel also sell to the world market? If so, how much will it sell?

(b) Now suppose that, at the request of domestic toy sellers, the government of Cuttenclip prohibits the cartel from selling to the domestic market at a price higher than it sells at on the world market. It can charge a domestic price higher than the world price of $4 only if it sells none to the world market. What now is the cartel’s optimal level of sales to the domestic and world markets and what price does it charge?
1. Christopher Carroll (QJE, 1994) has provided evidence that the uncertainty of future income has a strong negative effect on the household's willingness to increase consumption today in response to an increase in expected future income: the more uncertainty there is today about the size of a change in future income (of given mean size), the less the household changes its current consumption. Carroll argues that this finding rejects the certainty-equivalence version of the Permanent Income Hypothesis (PIH) usually discussed in the literature but is completely consistent with "cautious" behavior (sometimes called "prudence") by risk-averse households, which says that households consume less early in life than the PIH predicts and more later in life. Explain rigorously:

(A) The conditions that lead to "cautious" behavior by households.

(B) Why Carroll's finding supports cautious behavior and rejects the certainty-equivalence version of the PIH.

2. The AK model of endogenous growth is characterized by a production function of the form

\[ Y = AK \]

where A is a constant and K is physical capital. Note that capital has a constant rather than diminishing marginal product. Labor is not a factor of production. Households get utility from consumption C. Assume the utility function is CRRA:

\[ u(C) = \frac{C^{1-s} - 1}{1-s} \]

where s is the index of relative risk aversion. Capital grows with net investment, \( I = Y - C - dK \), where d is the depreciation rate.

One definition of a balanced growth path for the economy is a dynamic path along which everything that grows does so at a constant rate. The growth rates may differ from one variable to the next, but each growth rate is constant. (This definition implies that several "great ratios," such as the capital/labor ratio, are constant, but we will ignore that aspect of balanced growth here.) Optimal balanced growth rates are those consistent with social optimization (maximization of the representative agent's lifetime utility).

(A) Compute the optimal balanced growth rates for Y, C, and K in the AK model.

(B) "In the absence of shocks, the real rate of interest goes asymptotically to the rate of time preference." Show whether or not this assertion is true in the AK model.
3. In the fall of 1991, it was obvious that the United States was going to engage in a short war with Iraq sometime in the first half of 1992. In fact, such a war did happen and was over by the spring of 1992. Assume population and labor supply are fixed, utility is a function of consumption per person only, and that output is given by \( y = f(k) \), where \( f \) is concave and \( k \) is the capital/labor ratio. Assume also that the US started in steady state at the moment that people became aware of the coming war.

Use the Cass growth model (the standard optimal control solution for the centrally planned economy, with no technical progress) to explain the time path of real output in the US from the fall of 1991 to the end of 1992.

4. Consider the following model:

   (a) \( L = 2 \pi^2 + 4 (y_t - y^*)^2 \) \quad \text{disutility function of policy maker}
   
   (b) \( y_t = y_f + (\pi_t - \pi^*) \) \quad \text{aggregate supply function}
   
   (c) \( y = y_f + 3 \)

where \( \pi_t = \text{actual inflation} \); \( \pi^* = \text{public's expectation of inflation formed at time t-1} \);
\( y = \text{output} \); \( y_f = \text{natural level of output} \); \( y^* = \text{target level of output} \)

Assume that the policymaker can choose the level of inflation that minimizes his or her disutility.

a. Suppose the policymaker assumes that \( \pi^* = 0 \) initially. What level of \( \pi \) would be chosen?

b. Suppose agents are rational and know the complete model. What would inflation be in this case? Would the level of disutility be greater or smaller than for case (a)? Explain the intuition behind your answer.

c. Suppose that equation (b) changed to \( y_t = y_f + (\pi_t - \pi^*_t) + a_t \) where \( a_t \) is a supply shock that has an unconditional expectation of zero, is serially uncorrelated \( \text{i.e. } \text{Cov} (a_t, a_{t-1}) = 0 \), and has a variance of 9. The policymaker can observe \( a_t \) before deciding on \( \pi_t \), but the public cannot observe \( a_t \) when forming \( \pi^*_t \). What would be the choice of inflation and the level of disutility, again assuming the public has rational expectations? Compare this outcome to the case where the policymaker cannot observe \( a_t \) prior to choosing \( \pi_t \).

5. What is the importance to understanding the sources of business cycles fluctuations of the following:

a. the elasticity of labor supply with respect to the real wage rate
b. the covariance between real output and the real wage rate
c. the covariance between the real output and the price level
6. Consider the following model of inflation, attributed to Cagan:

\[ m_t - p_t = \alpha + \beta \pi_t^e + a_t \]  
\[ \pi_t = p_t - p_{t-1} \]  
\[ \pi_t^e = p_{t+1}^e - p_t \]

where \( m_t = \) log of nominal money supply, \( p_t = \) log of price level

\( \pi_t = \) inflation rate  
\( a_t = \) random error term with \( E(a_t) = 0 \) at time t-1

a. Solve for \( p_t \) under the assumption of adaptive expectations \( [\pi_t^e = \theta \pi_t + (1-\theta) \pi_{t-1}^e] \)

b. Solve for \( p_t \) under the assumption of rational expectations \( [\pi_t^e = E(\pi_t) \) information at time t], assuming that the money supply evolves according to

\[ m_t = \lambda_0 + \lambda_1 m_{t-1} + u_t \]  
where \( u_t \) has \( E(u_t) = 0 \) and \( Cov(u_t, u_{t-1}) = 0 \)

c. Discuss the policy relevance of these alternative expectations assumptions.
1. **Short Questions**

   (a) Queues in barber shops are longer in poor areas than in rich areas. Why?

   (b) True, False, or Uncertain. Explain your answer. If an industry's supply curve is upward sloping then an exogenous price increase will induce an increase in the output from all firms in the industry.

   (c) To reduce the price of potatoes, the government imposes a binding maximum price for tractors. Will the policy lower the potato price, raise it, or leave it unchanged? Explain.

   (d) When bad weather forces a commercial airline flight to divert from its intended destination, travelers incur inconvenience and extra costs due to unscheduled layovers. Airlines do not compensate travelers for such weather-related costs. Suppose that legislation is passed that requires airlines to compensate travelers fully for the expenses that they incur in these situations. Describe the effects on airline ticket prices, the quantities of airline tickets purchased, and the behavior of commercial airlines in stormy situations.

2. Given the consumer expenditure function, $\ln C(u, p_1, p_2) = \ln u + 0.4 \ln p_1 + 0.6 \ln p_2$, derive the consumer's Hicksian and Marshallian demand functions for $q_1$ and $q_2$. Show that these demand functions satisfy the homogeneity, symmetry, and adding-up restrictions.
3. The widgette industry is composed of two firms, Acme and Zartek. Each firm can produce any one of three output levels: 50, 75, or 100 units per month. As an industry, Acme and Zartek face a downward sloping demand curve for widgettes. Monthly industry profit is related to monthly industry output as in the following table:

<table>
<thead>
<tr>
<th>Industry output</th>
<th>Industry profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>32</td>
</tr>
<tr>
<td>125</td>
<td>35</td>
</tr>
<tr>
<td>150</td>
<td>30</td>
</tr>
<tr>
<td>175</td>
<td>21</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
</tbody>
</table>

Industry profits are distributed between the two firms in the same proportions as industry output. For example, if Acme produces 50 units and Zartek produces 100 units then total industry profit is 30, Acme earns one-third of industry profit (10), and Zartek earns two-thirds of industry profit (20).

(a) If Acme and Zartek are able to operate as a cartel, what output levels will they choose and what will be their profits?

(b) If Acme and Zartek determine their levels of output independently and simultaneously, what is the Nash equilibrium and what are the profits earned by the two firms in the Nash equilibrium?

(c) Is the Nash equilibrium a Pareto optimum? Why?

(d) Identify the Pareto optimal combinations of output.

(e) Now suppose that output is not determined simultaneously. Instead, Acme commits to a level of output and Zartek moves second. What is the Stackelberg equilibrium in this case and what profits do Acme and Zartek earn?

(f) Consider the Stackelberg equilibrium from part (e). If Acme cannot credibly commit to its announced level of output, what equilibrium will result? Is Acme better off if it is able to commit? Is Zartek better off if Acme can commit?
4. A firm producing computer chips produces 100 chips a day. It produces two types: high-quality chips and low-quality chips. The high-quality chips are used in name-brand computers and sell for a price of $P + \phi$. The low-quality chips can be sold to generic computer makers at the lower price of $P$. A fraction $\rho$ of a firm's output is high quality and the fraction can be determined by the use of a variable factor that increases quality. The effect of the quality-enhancing output is described by the following input requirement function:

$$f(\rho) = \frac{\gamma \rho^2}{2} \text{ for } 0 \leq \rho \leq 1.$$  

The parameter $\gamma$ is positive. The firm pays $w$ per unit of the quality-enhancing input.

(a) Derive an expression for $\rho^*$, the optimal quality choice of the firm. Is the expression homogeneous in any of its arguments?

(b) For a given $w$, higher values of the quality premium, $\phi$, induce higher levels of factor employment and increase profits. Let $\phi_L$ denote the highest value of $\phi$ at which a firm chooses $\rho^*=0$. Similarly, let $\phi_H$ denote the lowest value of $\phi$ at which a firm chooses $\rho^*=1$. What is the rate of change of firm profit with respect to $\phi$ at $\phi=\phi_L$? What is the rate of change of firm profit with respect to $\phi$ at $\phi=\phi_H$?

(c) Consider the same problem but without an explicit expression for input requirement. All you know is that input requirement is a function of the high-quality proportion. Denote the input requirement function as $f(\rho)$, with $f'(\rho)>0$. Prove that firm profit is a convex function of $\phi$ and that \[ \frac{\partial \rho^*}{\partial \phi} \geq 0 \].

5. Why do consumer prices typically fluctuate less than raw material prices in proportionate terms? For example, in the last year we have seen oil prices, primarily in response to a reduction in the supply of oil, rise much more rapidly than gasoline prices. Based on your knowledge of the relationship between factor and product markets, develop an economic model to explain the phenomenon that gasoline prices change proportionately less than oil prices for a given change in oil supply. Your model should include non-oil factors that are less than perfectly elastically supplied to the gasoline industry.
ANSWER FIVE (5) QUESTIONS.

1. In the spring of 1945, it became clear that the Second World War would end soon. In fact, the war did end in August of that year. Assume population and labor supply are fixed, utility is a function of consumption per person only, and that output per person is given by \( y = f(k) \), where \( f \) is concave and \( k \) is the capital/labor ratio. Assume also that the US was in general equilibrium (i.e., demand = supply in all markets) in the spring of 1945.

   Use the Cass growth model (the standard optimal control solution for the centrally planned economy, with no technical progress) to explain the time path of real output in the US from the spring of 1945 through December of 1945.

2. People may be uncertain about their future incomes. Consider a very simple endowment economy in which the household is given a random amount of income each period, with the distribution of income fixed over time and known to the household. The household chooses a path for consumption. The interest rate is constant and equal to the rate of time preference.

   (A) Set up the household optimization problem, stating the objective function and the budget constraint.

   (B) State the Euler condition for this problem.

   (C) Describe the time path of consumption. What does it depend on?

3. Consider the following model:

   \[
   \begin{align*}
   y_t &= m_t - p_t + v_t & \text{aggregate demand} \\
   y_t &= -\beta(w_t - p_t) + u_t & \text{aggregate supply} \\
   w_t &= \theta p_t + (1-\theta) p^e_t & \text{nominal wage equation} \\
   m_t &= m^* & \text{money supply equation}
   \end{align*}
   \]

   where \( y = \log \text{ of real output}; \quad w = \log \text{ of the nominal wage rate}; \)
   \( p = \log \text{ of price level}; \quad m = \log \text{ of money supply} \)
   \( p^e_t = \text{rational expectation of } p_t \text{ using information known at time (t-1)} \)

   The shocks \( v_t, u_t \) have zero means and constant variances, and are uncorrelated with each other. Agents know all the parameters of the model.

   a. Briefly discuss the rationale for equations (1)-(3).
   b. Derive the expressions for \( y_t \) and \( p_t \) in terms of \( m^* \) and the shocks.
   c. If you want to minimize the variance of \( y_t \), do you prefer \( \theta=1 \) or \( \theta=0 \)? Explain in words your result.
4. Consider an economy in which households choose consumption to maximize lifetime utility subject to their lifetime budget constraint. There is no uncertainty. For simplicity, suppose the labor supply and the capital stock are fixed.

Household i consumption demand: \( c^D_{it} = c^D(a_{it}, r_t) \)
\( a = \) wealth, \( r = \) interest rate

Household i money demand: \( h^D_{it} = h^D(c^D_{it}, r_t, \ldots) \)

Household i excess bond demand: \( b^D_{it} = b^D(r_t, y_{it}, a_{it}) \) \( y = \) household income

Household i income: \( y_{it}^S = A l^{1-\alpha} K^\alpha \)
\( l_t = \) labor, \( K_i = \) capital, both constant

Aggregate nominal money supply: \( M^S \)
constant

Suppose there is a permanent change in the world’s climate that lowers total factor productivity \( A \) (the coefficient in the production function). The change lowers \( A \) today and also is expected to lower \( A \) still more in the next period. After that, no further changes in are expected. Finally, suppose these expectations are correct.

Use the IS-LM-SS framework to explain the responses of

- aggregate consumption
- aggregate output
- real interest rate
- price level

in

(1) period 1
(2) period 2

Be sure to explain what you are doing (don’t just draw pictures) and to give the economic intuition for what happens in each period.

5. Some economists argue that a higher inflation rate increases the rate of economic growth, some argue that a higher inflation rate reduces the rate of economic growth, and some argue that economic growth is unaffected by the rate of inflation. Economic growth in this context is measured by the growth rate in per capita real output. Discuss these three positions with respect to the short-run and to the steady state, referring to any theoretical models and empirical evidence that you feel are relevant.

6. “The higher the inflation rate, the higher the amount of inflation tax collected.” Discuss this statement, including in your answer any assumptions you are making and any references to the literature you think are relevant.
1. Suppose an individual consumer's preferences are described by the following utility function:

\[ U = U(X,Y) = \log(X - A) + \log(Y - B) \]

Assuming that \( X > A \) and \( Y > B \), derive the Marshallian demand functions for \( X \) and \( Y \). Show that the demand functions for \( X \) and \( Y \) satisfy the zero-degree homogeneity, symmetry, and adding-up restrictions of consumer demand.

2. Journal articles are produced using computers and labor. Suppose that because of innovations in the computer industry the cost of computing falls by 5 percent. Assuming that the market for professional journal articles is competitive, what would you expect to happen to prices and quantities in the markets for labor and journal articles if the price elasticity of demand for journal articles is \(-1.5\), the elasticity of substitution between computing and labor is zero, the cost share of computing in journal articles is 0.4, the elasticity of supply of labor is 0.3, and the supply of computing services to the journal article industry is infinitely elastic? Indicate intuitively why the numerical estimates are as they are calculated.

3. A firm produces a single output using a single input. Denote the quantity of the output as \( y \) and the quantity of the input as \( x \). The production function of the firm is \( y = x^{1/2} \). The firm has no influence on \( w \), the price that it pays for the input.

   (a) The government institutes a subsidy program that increases the output price received by the firm from \( p=1 \) to \( p=8 \). Assuming that \( w=1 \), and that \( w \) is unaffected by the subsidy, what will be the change in the firm's profits?

   (b) Now suppose that the industry is composed of 100 firms that are identical in all respects to the firm analyzed in part (a). The industry is competitive and faces an input supply curve of \( x = 25 w \). If the government institutes a subsidy that increases the price received by all of the firms in the industry from \( p=1 \) to \( p=8 \), what will be the change in a single firm's profits?

   (c) Apart from consumers of the industry's output and the 100 producers, whose welfare is affected by the subsidy in part (b) and by how much is it affected?
4. A bar owner on Hillsborough Street faces two distinct sets of customers: faculty and students. Each student has an inverse demand curve for beer given by:

\[ p_s = 16 - q_s. \]

Each faculty member has an inverse demand curve for beer given by:

\[ p_F = 8 - q_F. \]

There are 100 students and 100 faculty members. The marginal cost of bottles of beer to the bar is constant at $0.50/bottle.

(a) If the bar is restricted to charging a single price per bottle, what is the optimal price for the bar to charge and what would be its profits (counting only beer costs)?

(b) If the bar is able to identify costlessly which of its customers are students and which of its customers are faculty, what are the optimal prices to charge students and faculty? What would be the bar’s profits if it charged those prices?

(c) If the bar can charge an entry fee to its customers, and can charge different entry fees and different unit prices for beer to each type of customer, what entry fees and per-unit prices will it charge? What would be its profits under this pricing scheme?
Macroeconomics Preliminary Examination
July 2001

ANSWER FIVE (5) QUESTIONS

1. Consider the following traditional IS-LM model of the economy:

   (i) \( Y_t = C_t + I_t + G_t \)
   (ii) \( C_t = a_0 + a_1(Y_t - T_t) - a_2r_t \)
   (iii) \( I_t = b_0 - b_1r_t \)
   (iv) \( G_t = \text{G}^* \)
   (v) \( T_t = d_0 + d_1Y_t \)
   (vi) \( M_t'/P_t = M^*/P_t \)
   (vii) \( m^d_t = vY_t - gr_t \)

   where

   \( Y \) = real output
   \( C \) = real consumption
   \( I \) = real investment
   \( G \) = real government purchases
   \( T \) = real tax payments
   \( M^* \) = nominal money stock
   \( m^d \) = real money demand
   \( P \) = price level
   \( r \) = real interest rate
   \( a, b, c, d, v, g \) = constants
   \( M^*, G^* \) = exogenous values

   Explain what changes you feel are necessary to bring this model into conformity with modern macroeconomic theory and with the existence of a banking system. (Obviously, the model is linear, whereas the real world is non-linear. Ignore that problem in your discussion and treat the model as a linear approximation to the non-linear real world.)

2. Explain the following famous fact:

   Suppose we collect cross-section data on households. We graph current income on the horizontal axis and current consumption on the vertical axis. Typically the slope of this line is clearly less than one (that is, the line is flatter than a 45 degree line) and has a positive intercept on the vertical axis. In contrast, if we collect time series data for the economy and make the same graph, the slope of the resulting line is close to one (that is, it is close to a 45 degree line) with an intercept at zero.
3. Suppose the economy of Lower Slobovia is described by the following mathematical model. The aggregate production function is

\[ y_t = f(k_t) \quad \text{with} \quad f' > 0 > f'' \]

where \( y \) = real output per person and \( k \) = capital/labor ratio. The law of motion for \( k \) is

\[ \frac{dk_t}{dt} = f(k_t) + F - c_t - (\delta + n) k_t \]

where \( c \) = consumption per person, \( \delta \) = rate of depreciation, and \( n \) = rate of population growth and \( F \) = foreign aid per person received from another country. Society seeks to maximize

\[ V = \int_0^\infty u(c_t) e^{-\rho t} \, dt \]

where \( u \) is the concave utility function of the representative agent \( (u' > 0 > u'') \) and \( \rho \) is the rate of time preference.

Suppose all Lower Slobovians (affectionately called Slobs in the rest of the world) suddenly learn that their foreign aid payments will cease one year from now. They expect the payments never to resume. Assuming the Lower Slobovian economy started in steady state, derive and explain its optimal dynamic adjustment toward the new steady state. Be sure to describe the time paths of \( y \), \( c \), and \( k \).

4. Explain and critique the theory behind the prediction that there is an "inflation bias" to monetary policy. Describe and evaluate two mechanisms that might exist to reduce or eliminate this bias? Explain your reasoning and feel free to refer to any papers in the literature you think are relevant.

5. Discuss the overlapping-generations model of money in which people live for two periods but only work in the first period. Assume that goods cannot be stored, that there is no capital, and that each generation is \( n \) percent greater in number than the previous generation. Under what conditions will money have value in this model? What is the optimal growth rate of money in this model and why?
6. Consider the following model:

\[ y_t^d = m_t - p_t + \varepsilon_t \]  
aggregate demand

\[ y_t^s = \beta(p_t - p_t^*) + \gamma y_{t-1} + u_t \]  
aggregate supply (\( \beta > 0 \))

\[ y_t^d = y_t^s = y_t \]  
short-run equilibrium

where
\begin{itemize}
  \item \( y \) = deviation of output from its steady-state equilibrium
  \item \( m \) = money supply
  \item \( p \) = price level
  \item \( p_t^* \) = public's expectation of \( p_t \) formed at time \( t-1 \)
  \item \( \varepsilon_t \) and \( u_t \) are serially uncorrelated error terms with expected values of zero at time \( t \)
\end{itemize}

a. Equation (2) is often referred to as a Lucas supply curve. According to Lucas, why is \( \beta \) positive and what does its value depend on?

b. Assume that \( m_t \) is set by the central bank at a constant level equal to \( m^* \), which is known by the public. Assume also that the public knows the value of all variables dated at time \( t-1 \) or earlier and forms rational expectations. What are the short-run equilibrium values of \( y_t \) and \( p_t \)?

c. Suppose that the central bank can observe \( \varepsilon_t \) before setting the value of \( m_t \), but the public only knows \( m^* \). The central bank sets \( m_t = m^* + \nu_t \) by choosing \( \nu_t \) after observing \( \varepsilon_t \). It cannot observe \( u_t \) at the time it sets \( m_t \). How should it choose \( \nu_t \) if it wants to minimize \( E(y_t^d) \)? Discuss your answer.
Answer all six questions.

1. The (logarithmic) expenditure (or cost) function of an individual consumer is as follows:

\[ \log C(p_1, p_2, U) = a(p_1, p_2) + U \cdot b(p_1, p_2), \]

where \( a(p_1, p_2) = \alpha_0 + \alpha_1 \log p_1 + \alpha_2 \log p_2, \) and

\[ b(p_1, p_2) = \beta_0 p_1^{\beta_1} p_2^{\beta_2}. \]

The parameters satisfy the restrictions:

\[ \alpha_1 + \alpha_2 = 1, \quad \beta_1 + \beta_2 = 0. \]

(a) Derive the Marshallian demand functions.

(b) Verify that the homogeneity and symmetry conditions are satisfied.

(c) Verify that the Hicksian (compensated) demand curves are negatively sloped.

2. Soybeans are grown in the country of Nebonia and are crushed into the joint products of soybean oil and soybean meal. The oil is used in cooking and the meal is fed to animals. Each pound of soybeans yields one cup of oil and one cup of meal.

(a) Analyze the effect of a sales tax on soybean oil. Include in your discussion the effects of the tax on consumers of oil, consumers of meal, and soybean farmers. In each instance, identify appropriate measures of the gains and losses.

(b) Suppose that the market for beans, oil, and meal is characterized by the following: the elasticity of demand for oil is \(-3/2\), the elasticity of demand for meal is \(-3\), the elasticity of supply of soybeans is 3, and the equilibrium price of a cup of oil is equal to \(1/2\) of the equilibrium price of a pound of soybeans. Calculate the percentage change in the price of soybeans that would result from a 10% sales tax on soybean oil.
3. Wordish is a company that develops word processing software and Spreader is a company that develops spreadsheet software. In isolation from each other they are deciding what file format to feature in their software. Each firm has the choice of featuring format X or format Y. Because computer users value being able to read spreadsheets into their word processors, they value file formats that are compatible. Further, Wordish has a preference for format X and Spreader has a preference for format Y. Therefore, if both firms adopt format X, which is preferred by Wordish, the profit to Wordish is 200 and the profit to Spreader is 100. On the other hand, if both firms adopt format Y (the format preferred by Spreader) the profit to Spreader is 200 and the profit to Wordish is 100. If they fail to coordinate, and adopt different file formats, then each firm has a profit of -100.

   (a) Set up the problem as a non-cooperative game and identify the Nash equilibria, if any, in pure strategies.

   (b) Define a mixed strategy and identify any Nash equilibria in mixed strategies.

   (c) Rank the equilibria you found in parts (a) and (b) by the Potential Pareto Improvement (economic efficiency) criterion.

4. An industry is dominated by a firm that is responsible for three-fourths of the industry's output. The rest of the industry's output comes from a large number of price-taking firms. The market-level elasticity of demand is -1. The supply elasticity from the price-taking firms is 4.

   (a) What is the elasticity of demand facing the dominant firm?

   (b) What is the percentage markup of price over marginal cost for the dominant firm? (The percentage markup is called the Lerner index.)

   (c) If one adopts the Lerner index as a measure of the harm due to non price-taking behavior, explain why the market share of the dominant firm can be a misleading measure of the extent of market power.
5. Suppose that the profit function of an individual firm in the gasoline industry has the form:

\[ \pi = \pi(P_g, P_o), \]

where \( P_g \) is the price of gasoline and \( P_o \) is the price of oil. Assume that the industry is presented with an exogenous increase in the price of oil and you are asked to evaluate the impact on profit from this increase. Suppose for sake of simplicity that the supplies of other factors to the industry are perfectly elastic.

(a) Evaluate the effect on profit assuming that the industry is competitive and that the profit function above is representative of all firms in the industry. That is, suppose that \( P_g \) is endogenously determined by supply and demand for gasoline so that \( P_g = P_g(P_o) \). What are the conditions under which profit will decrease or increase?

(b) Evaluate the effect on profit from the increase in the price of oil, now assuming that the industry is a monopoly. If the (inverse) demand facing the firm has the form \( P_g = f(Q_g) \), where \( Q_g \) is the quantity of gasoline demanded, then the expression for profit for the monopolist is given as:

\[ \pi = P_g Q_g - C(Q_g, P_o) = f(Q_g)Q_g - C(Q_g, P_o), \]

where \( C(\cdot) \) is the monopolist's cost function. Assume that the firm behaves as a monopolist in the market for gasoline but is a price taker in the market for oil. Is the result the same as in a competitive industry?

6. Consider an isolated economy with 100 identical individuals producing and consuming two goods, \( X \) and \( Y \), using two inputs \( K \) (capital) and \( L \) (labor). The production functions for \( X \) and \( Y \) are:

\[ x = \min(K, L) \text{ and } y = \min(K, L/4). \]

Each individual derives utility only from the consumption of \( X \) and \( Y \) and has the utility function \( U = xy \), where lower-case letters denote per capita consumption. The total factor supplies are \( K = 600 \) and \( L = 1200 \).

(a) Draw the economy's production possibilities Frontier (PPF). Are all points along the PPF points of full employment of \( K \) and \( L \)?

(b) Calculate the competitive equilibrium quantities of \( X \) and \( Y \) produced and the equilibrium per capita consumption quantities \( x \) and \( y \).

(c) Let \( Y \) be the numeraire, i.e., \( P_Y=1 \). Find the equilibrium \( P_X \).

(d) Find the equilibrium prices of labor and capital.
Do any five of the following six questions. All questions have equal weight in determining your grade.

1. Several studies of the consumption function report evidence of "excess sensitivity" of aggregate consumption to aggregate transitory income, meaning that the parameter $c$ is statistically significant in a regression such as

$$C_t = a + bY^p_t + cY^T_t + dX_t + e_t$$

where $Y^p$ and $Y^T$ are permanent and transitory income, $X$ is a vector of other explanatory variables, and $e$ is the residual.

Discuss the issues you would consider (other than those dealing with the details of the particular estimation method used) in evaluating such evidence.

2. Suppose that the aggregate production function is:

$$y_t = f(k_t), \quad f' > 0 > f''$$

where $y =$ real output per capita and $k =$ capital/labor ratio. The law of motion for $k$ is

$$\frac{dk_t}{dt} = f(k_t) - c_t - (\delta + n)k_t$$

where $c =$ consumption per capita, $\delta =$ rate of depreciation, and $n =$ rate of population growth. Society seeks to maximize

$$V = \int u(c_t)e^{-\rho t}dt$$

where $u$ is a concave utility function ($u' > 0 > u''$) and $\rho$ is the rate of time preference.

A. Set up the maximization problem, stating the Hamiltonian and the necessary conditions.

B. Derive and draw the phase diagram, showing the equilibrium loci and the dynamic adjustment paths for the state and costate variables.

C. Solve for the steady-state values of capital and gross investment.
D. Edmund Phelps, using the Solow growth model, derived the "Golden Rule" value of $k$ to be the value $k^*$ such that

$$f'(k^*) = (n+\delta)$$

This is the value of $k$ that maximizes $c$ in the Solow growth model. Compare it to the value obtained in part C of this question and explain the economic meaning of any differences.

3. The Japanese economy has been doing badly for several years. The Japanese government recently undertook a fiscal program to stimulate the economy. The plan increased government purchases on "public works projects" and gave lump-sum tax rebates to all taxpayers for two years. The government deficit consequently was increased.

Evaluate the effects of this fiscal program's three parts (purchases, taxes, debt) on

- $Y$: gross domestic product
- $C$: aggregate consumption
- $I$: investment
- $L$: labor supply
- $r$: real interest rate
- $P$: price level

Assume the public works projects have no productive value (apparently a good guess).

4. Consider the following model of an economy:

$$r_i = -\beta (w_i - p_i) + \varepsilon_i$$  

labor demand  \hspace{1cm} \beta > 0$$

$$r_i' = 0$$  

labor supply (normalized)$$

$$w_i = w_i' + \phi$$  

nominal wage set by monopoly union  \hspace{1cm} \phi > 0$$

$$w_i' = p_i'$$  

nominal wage expected to clear the labor market$$

$$\pi_i = p_i - p_{i-1}$$  

inflation (under the control of the central bank)$$

$$\pi_i' = p_i' - p_{i-1}$$  

expected inflation$$

$$W_i = l_i^2 + \lambda \pi_i^2$$  

Loss function of central bank  \hspace{1cm} \lambda > 0$$

$l = \log$ of labor  \hspace{1cm} $w = \log$ of nominal wage rate  \hspace{1cm} $p = \log$ of price level

$\varepsilon_i =$ shock to labor demand that has expected value zero and constant variance $\sigma^2$

In this model, a monopoly union can set the nominal wage above the wage that would be expected to clear the labor market.

a. What would the central bank choose for $\pi$ if the union had rational expectations and no one could observe $\varepsilon_i$ at the time the nominal wage rate was set? Discuss how your answer depends on $\beta$ and $\lambda$. 
b. Suppose that the central bank can observe \( \varepsilon_t \), but the union cannot and that the central bank wants to set up a rule for inflation that minimizes the expected loss function, that is it wants an equation for inflation that depends on \( \varepsilon_t \) and the parameters of the model. What equation would it choose? How will the expected loss compare to the expected loss from setting inflation according to part (a)?

5. The U.S. Treasury recently announced that it would no longer issue 30-year bonds and that the longest maturity bond that it would now issue would be the 10-year bond. The Treasury argued this would lower the expected interest cost of the national debt.

a. Under what conditions would you agree with the Treasury's analysis and under what conditions would you disagree?

b. Under what conditions would the Treasury's action alter the present value of taxes paid by households?

6. Short-run stochastic models of the economy usually have three categories of shock: productivity or supply shocks, financial shocks, and spending shocks. Suppose that the central bank is considering an interest rate target or a money supply target as the intermediate target for monetary policy and that the sole goal of policy is to stabilize output about its full employment level. How does the relative performance of these two targets depend on the relative prevalence of the different types of shocks? Explain you reasoning.
ECONOMICS CORE PRELIMINARY EXAMINATION
NORTH CAROLINA STATE UNIVERSITY

Microeconomics
Wednesday, January 2, 2002

Answer all six questions using the writing pads provided. Write on one side of the paper only and start each question on a new page. Questions are weighted equally.

In questions where the answer depends on particular assumptions, be specific as to the assumptions to make. Your answers should demonstrate your command of economic reasoning.

1. Consumer Theory

For the utility function, \( u = x_1^{1/4} x_2^{1/2} \), derive each of the following, where \( p_1, p_2 \) are prices, \( x_1, x_2 \) are quantities, \( u \) is utility, and \( I \) is income:

a. Marshallian (ordinary) demand functions: \( g_i(p_1, p_2, I) \)
b. Indirect utility function: \( \psi(p_1, p_2, I) \)
c. Expenditure function: \( C(p_1, p_2, u) \)
d. Hicksian (compensated) demand functions: \( h_i(p_1, p_2, u) \)

2. Information Economics

George A. Akerlof, A. Michael Spence and Joseph E. Stiglitz won the Nobel prize in economics this year for their work on the economics of information. One specific research area the three scholars worked on is adverse selection. Describe the contributions of at least two of the above Nobel prize winners in this area.

3. Cartel Theory

An agricultural industry acts as a cartel and jointly maximizes profit. Suppose that it also taxes its producers by the amount \( T = tQ \) (\( t = \) per unit tax, \( Q = \) output) and can spend the money on either advertising, \( A \), or research, \( R \). The inverse demand function facing the industry is \( D(Q,A) \) where \( \partial D/\partial Q < 0, \partial D/\partial A > 0 \). Also, the firm incurs costs of production equal to \( C(Q,R)+A \) where \( \partial C/\partial Q > 0, \partial C/\partial R < 0 \). Assuming that the tax revenue can be spent on only advertising or research \( (T = A+R) \),

a. Write out the expression for constrained profit maximization.
b. Derive the first order conditions for profit maximization with respect to Q, A, R, and \( \lambda \) (the Lagrangian multiplier).

c. Give an economic interpretation of the first-order conditions. What is the opportunity cost of spending an additional dollar on advertising?

4. General Equilibrium and Pareto Efficiency

Consider an economy with two firms, one consumer, and three goods: labor \( n \) and two produced goods \( q_1 \) and \( q_2 \). Firm 1's production function is \( q_1 = n_1 \). Firm 2's production function is \( q_2 = q_1 n_2 \), and it therefore has an externality with \( q_1 \). The consumer's utility function is \( u = q_1 q_2 \), and the endowment of labor is \( 1 \). Let the price of labor be \( 1 \).

a. Find the competitive equilibria for the economy (if any).

b. Find the Pareto efficient allocations for the economy (if any);

c. The government wishes to obtain Pareto efficiency by putting an excise tax \( t \) on good 2 and giving the money collected back to the consumer as a lump-sum subsidy \( S \). What \( t \) and \( S \) should be used?

5. Derived Demand

The mowing of lawns requires only labor (gardeners) and capital (lawn mowers) and one lawn can be mowed per hour. These factors must be used in the fixed proportions of one worker to one lawn mower, and production exhibits constant returns to scale. Suppose that the wage rate of gardeners is \$10 per hour and that lawn mowers rent for \$20 per hour and that the price elasticity of demand for mowed lawns is \(-2 \). Also assume that both labor and lawn mowers are in sufficient supply that changes in demand for labor and lawn mowers would have no perceptible effect on wages or rental rates.

a. What is the equilibrium price of mowed lawns? What is the elasticity of demand for labor with respect to a change in the wage rate? The elasticity of demand for lawn mowers with respect to a change in rental rate of lawn mowers?

b. Suppose that the government is concerned about the amount of air pollution created by lawn mowers and implements a per unit tax of \$1 for every hour rented. What would you expect to happen to the number of lawns mowed, the number of workers hired, and the number of lawn mowers rented? State your answers in terms of percentage change from the initial equilibrium quantities.
6. **Contract Theory**

Consider a hidden action model in which the owner is risk-neutral while the manager has preferences defined over the mean and the variance of his income $w$ and his effort level $e$ as follows: Expected utility $= E(w) - \phi \text{Var}(w) - g(e)$, where $g'(0) = 0$, $(g'(e), g''(e), g'''(e)) > 0$ for $e > 0$, and $\lim_{e \to -\infty} g'(e) = \infty$. The manager has a reservation utility which is normalized to zero. Possible effort choices are $e \in \mathbb{R}_+$. Conditional on effort level $e$, the realization of profit $\pi$ is normally distributed with mean $e$ and variance $\sigma^2$.

(i) Restrict attention to linear compensation schemes $w(\pi) = a + b\pi$. Show that the manager’s expected utility given $w(\pi)$, $e$ and $\sigma^2$ is given by $a + b e - \phi b^2 e^2 - g(e)$.

(ii) Derive the optimal contract when $e$ is observable.

(iii) Derive the optimal linear compensation scheme when $e$ is not observable. What effects do changes in $\phi$ and $\sigma^2$ have, and why?
Answer any five (5) of the following six questions. Each question is worth 20% of your exam grade.

(1) Most central banks implement monetary policy by setting a target for a short-term, nominal interest rate. That target is changed periodically as economic conditions change.

i. Explain why raising the interest rate target reduces future inflation.

ii. Declining unemployment rates are thought by some commentators to indicate the need to raise the interest rate target. Explain the logic of this conclusion and evaluate the argument.

iii. Some commentators argue that raising the interest rate target will cause economic activity to decline. Explain the logic of this conclusion and evaluate the argument.

iv. What determines the appropriate steady-state value of the interest rate target and what is the result of consistently setting the target below this value?

(2) Consider an economy described by the following functions:

Production function

\[ Y_t = AK_t^\alpha L_t^\beta \]

where \( Y \) = aggregate output, \( K \) = aggregate capital stock, \( L \) = total labor force = population, \( 0 < A \) (constant), \( 0 < \alpha < 1, \ 0 < \beta < 1 \)

Intertemporal utility function

\[ \int_0^\infty u(c_t)e^{-\rho t}dt = \int_0^\infty \left( \frac{c_t^{1-\eta} - 1}{1 - \eta} \right) e^{-\rho t}dt \]

where \( C \) = aggregate consumption, \( c = C/L \) = consumption per person, \( 0 < \rho \) (constant)

Aggregate budget constraint

\[ Y_t = C_t + I_t \]

where \( I \) = gross investment

Capital accumulation

\[ \frac{dK_t}{dt} = I_t - \delta K_t \]
where $\delta = \text{depreciation rate, constant}$

Population growth

$$\frac{dL_t}{dt} = nL_t$$

where $0 < n$ (constant)

Suppose the economy initially is on its balanced growth path when government is unexpectedly invented. The new government imposes a proportional income tax $T_t = \tau Y_t$ and uses the revenue to pay for economically useless purchases $G_t$. The marginal tax rate $0 < \tau < 1$ is constant.

Explain how this change in fiscal policy affects the growth rate of income per person $Y/L$ on its balanced growth path (i.e., ignore transition effects) in each of the following two models:

(A) Cass growth model: $\beta = 1 - \alpha$, so that $\alpha + \beta = 1$.

(B) Endogenous growth model: $\alpha = 1$, $0 < \beta < 1$, so that $\alpha + \beta > 1$.

Explain any differences between the results of the two models.

(3) Frydman and Rappoport (AER, Sept. 1987) present evidence that, in terms of the effect on real economic activity, there is no significant difference between a change in the nominal money supply that is anticipated and one that is not.

Consider an equation of the form

$$y_t = y_t^* + \alpha(L)(M_t - E_{t+1}(M_t)) + \beta(L) E_{t+1}(M_t) + u_t,$$

where $y$ is real output, $y^*$ is equilibrium real output, $M$ is the nominal money supply, $u$ is random disturbance and $\alpha(L)$ and $\beta(L)$ are polynomials in the lag operator, e.g., $\alpha(L) = \alpha_0 + \alpha_1 L + \ldots + \alpha_m L^m$, and $L$ is the lag operation, e.g., $L(x_t) = x_{t-1}$, $t$ indexes calendar quarters.

Frydman and Rappoport could not reject the hypothesis the $\alpha(L) = \beta(L)$ implying that the (1) could be written as

$$y_t = y_t^* + \theta(L)(M_t) + u_t,$$

where $\alpha(L) = \beta(L) = \theta_0 + \theta_1 L + \ldots + \theta_m L^m$.

(i) Discuss the extent to which $\alpha(L) = \beta(L)$ is consistent with the following three types of models:

a. Intertemporal, optimizing, general equilibrium models with perfectly flexible
prices and wages.

b. Lucas imperfect information model.

c. New Keynesian wage contracting models where nominal wages are predetermined for two or more periods.

(ii) Friedman and Rappoport decisively rejected the hypothesis all the coefficients $\theta_i$ are zero; that is, they rejected the null hypothesis that $\theta_0 = \theta_1 = \ldots = \theta_m = 0$. A different question, however, is the value of the sum of these coefficients: $\sum \theta_i$. Does theory imply anything about this sum? Discuss.

(4) Assume that the representative agent’s utility function is

$$U_t = \ln c_t + \ln m_t$$

where $U$ designates utility, $c$ is consumption and $m$ is real money balances and $\ln$ designates natural logarithms. Walsh shows that the first-order condition involving money can be reduced to

$$U_m / U_c = i / (1 + i)$$

where $U_m$ and $U_c$ are partial derivatives and $i$ is the nominal interest rate on a one-period bond. Assume the rest of the model satisfies the conditions for super-neutrality of money.

(a) How is the steady-state value for real money balances related to the steady-state rate of growth of the nominal money supply? Explain.

(b) Is faster nominal money growth preferred to slower nominal money growth? Explain.

(5) Suppose the economy experiences a temporary negative productivity shock. For example, if the production function is $Y_t = A_t K_t \cdot L_t^{1-\alpha}$, then a simple type of negative productivity shock is a reduction in the coefficient $A_t$. Let us suppose that a "temporary shock" means one that is expected to disappear at a specific time in the near future (perhaps a year away). It often is argued that fiscal policy can be used to counteract such a cyclical shock. Suppose that government purchases $G_t$ are exogenous and that taxes are a simple proportional income tax:

$$T_t = \tau_t (Y_t - E)$$

where $\tau$ is the marginal tax rate and $E$ is exemptions. The government budget constraint is

$$G_t + r_t B_t = T_t + dB_t / dt$$

where $r$ is the interest rate and $B$ is the stock of government debt. The government has, then, four fiscal policy tools: $G$, $\tau$, $E$, and $B$; but only three of these are independent because of the
government budget constraint. To be specific, let us treat $G$, $\tau$, and $E$ as the three independent policy tools.

Use the continuous-time Cass growth model to give a rigorous explanation of the usefulness of these policy tools in combating a negative productivity shock. Discuss each of the three policy tools separately, not in combination with either of the other two. For simplicity, assume labor supply $L$ is constant (no growth, no labor/leisure choice).

(6) Imagine that you are reviewing grant proposals for the National Science Foundation. An applicant proposes estimating investment demand functions for the different kinds of capital used in some particular industry. He proposes the following model:

$$I_j(t) = a_j [K_j^*(t) - K_j(t-1)]$$

$$K_j^*(t) = b_0 + b_1 Q^N_t$$

$$Q^N_t = (\sum_{i=1}^{4} Q_{i,t})/4$$

where $j = 1, ..., J$ indexes the type of capital, $t$ is the time period, $a_j$ is constant for each $j$, $K^*$ is desired capital stock, $Q$ and $Q^N$ are current and "normal" (that is, expected) levels of sales. There are $J$ types of capital in the industry being studied, and the applicant proposes to estimate each pair of the $J$ investment and desired capital stock equations separately. He will use quarterly data.

Evaluate this proposal, explaining any difficulties you see in it, citing both theory and evidence to support your conclusions.
1. Two companies, Acme and Bacme, produce similar tasting Colas. Each is considering its advertising strategy for the coming quarter. Each firm has two strategies to consider: pursue an aggressive negative advertising campaign that denigrates the product produced by the other firm or pursue an upbeat, gauzy campaign that does not mention or otherwise refer to the competitor's product. Both companies have a preference for "going negative." Specifically, if Acme goes negative while Bacme stays positive, Acme will earn a profit of 400 and Bacme will earn a profit of only 100. Likewise, if Bacme goes negative while Acme stays positive, Bacme will earn a profit of 400 and Acme will earn a profit of only 100. If both firms go negative each will enjoy an operating profit of -700. Finally, if both firms use a positive advertising campaign each will earn a profit of 200.

(a) Set up the problem as a non-cooperative game and identify any and/or all pure strategy Nash equilibria.

(b) Define a mixed strategy and identify any and/or all Nash equilibria in mixed strategies.

2. The indirect production function is obtained by maximizing output subject to an expenditure constraint. That is,

\[
y(r_1, r_2, C) = \max_{\{x_1, x_2\}} \left[ f(x_1, x_2) \right] \, r_1 x_1 + r_2 x_2 = C
\]

is the indirect production function where \( r_1 \) (\( r_2 \)) is the unit price of input \( x_1 \) (\( x_2 \)), \( C \) is total expenditure on inputs, and \( f(x_1, x_2) \) is a production technology that satisfies standard economic requirements.

(a) What general properties would you ascribe to \( y(r_1, r_2, C) \)? Use duality properties to derive unconditional (Marshallian) factor demands of the form \( x^*_i (r_1, r_2, C) \) from the indirect production function.
(b) Assume the indirect production function is given by 
\[ y(r_1, r_2, C) = \frac{C - a(r_1, r_2)}{b(r_1, r_2)} \]
where \( a(r_1, r_2) = \alpha_1 r_1 + \alpha_2 r_2 \) and \( b(r_1, r_2) = \beta_1 r_1^\beta_1 r_2^\beta_2 \), \( \beta_1 + \beta_2 = 1 \). Derive the unconditional (Marshallian) factor demand equations and the conditional (Hicksian) factor demand equations. Recall that conditional factor demands are of the form \( x_i^e(r_1, r_2, y) \).

(c) Verify that the compensated (Hicksian) factor demands in part (b) satisfy homogeneity of degree zero in prices. Also, verify that the Hicksian factor demands are symmetric.

3. Shorter Questions

(a) “A firm’s output decision is not normally affected by a lump-sum tax on each producer in an industry, so equilibrium industry output cannot be affected by such a tax.” True or false. Justify your answer.

(b) Suppose two individuals face the same prices, have the same income, and have identical tastes yet choose different consumption bundles. Does this necessarily imply one of the consumers is irrational? Explain.

(c) In the classic externality problem where the rancher-producer raises cattle (who invariably trample some of a neighboring farmer’s crop), unless the rancher is somehow held liable for the crop damage cattle would be produced where price equals marginal private cost of cattle. True or false. Explain.

4. Wool (W) and mutton (M) are jointly produced from sheep (S) according to the production function

\[ S = \max (W, M) \]

The demand curves for wool and mutton are

\[ P_w = 5 - W \]

\[ P_m = 2 - M \]

where \( P_w \) is the price of wool and \( P_m \) is the price of mutton. The supply curve for sheep (\( P_s \) is the price of sheep) is

\[ P_s = 3 + S \]

(a) What is the supply curve of mutton? What is the supply curve of wool?

(b) Find the equilibrium prices and quantities for wool and mutton.
(c) Suppose as a result of trade restrictions imposed by the United States on lamb imported into the U.S. the demand for mutton increases by $1 per unit (lamb and mutton are substitutes in consumption). What is the effect of the increase in demand for mutton on prices and quantities of mutton and wool?

5. It has been asserted that the monopolist is different from a competitive firm in that output price increases by more than the increase in its costs. Evaluate this proposition assuming that the monopolist’s profit function is as follows:

\[ \pi = p(q)q - cq \]

where \( p \) is price, \( p(q) \) is the monopolist’s demand curve, \( q \) is output, and \( c \) is the constant marginal cost. Specifically,

(a) Calculate the effect of a change in cost \( (c) \) on price \( (p) \).

(b) Evaluate the magnitude of the comparative static result, \( \frac{dp}{dc} \), and determine the conditions in which it is larger than 1. (Hint: consider in your evaluation the two cases of a linear demand curve and a constant elasticity demand curve.)

6. Stella is a delta cotton farmer with initial endowment of monetary wealth \( W_0 \). Stella is also a von Neumann-Morgenstern decision maker and as such she is strictly risk averse, that is, her utility function \( U \) is strictly concave, with \( U'' < 0 \). Assuming there is no springtime flood, Stella can earn an annual income from raising cotton of \( y \) with probability \( \pi \), \( 0 < \pi < 1 \). With probability \( 1 - \pi \) it floods, and Stella earns no cotton income. Her utility depends only on the terminal wealth (i.e., initial wealth plus additional funds obtained) that she has available with which to consume goods at the end of the crop year. Assume that Stella has no ability to influence the probability of whether or not a flood occurs. Stella may purchase private disaster insurance. Insurance companies allow individuals to select the payment received in the event of a flood. For each dollar the insurance firm pays to an individual in the event of a flood, the individual pays a premium of \( p \) if a crop is raised.

(a) Assume that it is optimal for Stella to purchase some insurance. State the first-order condition for an optimum.

(b) Determine the actuarially fair premium, \( p \). Determine the optimal disaster insurance payment \( y^d \) when the premium is actuarially fair.
ECONOMICS CORE PRELIMINARY EXAMINATION
NORTH CAROLINA STATE UNIVERSITY

Microeconomics
Wednesday, January 8, 2003

Answer all six questions using the writing pads provided. Write on one side of the paper only and start each question on a new page. Questions are weighted equally.

In questions where the answer depends on particular assumptions, be specific as to the assumptions to make. Your answers should demonstrate your command of economic reasoning.

1. Short questions:

a) While trying to buy car insurance, Mr. Niki Lauda had been offered an expensive contract with full insurance and a cheaper contract with a $500 deductible. Consulting with his wife on the phone, he explained that insurance companies offer contracts with deductibles since that way they make the drivers who suffer many accidents pay $500 each time. Is Mr. Lauda’s argument reasonable from the adverse selection point of view? Why? Why not?

b) Sally consumes only beer and beans, and she always consumes them in fixed proportions. What is her income elasticity of demand for beer? For beans? What is the compensated (Hicksian) cross-price elasticity of demand between beer and beans? What are simple expressions for the Marshallian own-price elasticities of demand? Do her Hicksian demands satisfy the negativity requirement? Briefly explain/discuss your results.

2. The translog cost function is said to be a (local) second-order approximation to an arbitrary cost function. It is defined as follows:

\[
\ln C(r_1, r_2, y) = \alpha_0 + \sum_{i=1}^2 \alpha_i \ln r_i + \frac{1}{2} \sum_{i=1}^2 \sum_{j=1}^2 \gamma_{ij} \ln r_i \ln r_j + \sum_{i=1}^2 \beta_i \ln r_i \ln y + \phi \ln y + \frac{1}{2} \tau (\ln y)^2
\]

where \( r_i \) are unit prices for factors, \( i = 1, 2 \), and where \( y \) is output. Assume that \( \sum_{j=1}^2 y_j = 0 \), \( i = 1, 2 \).

a) Derive the logarithmic version of Shephard’s lemma. That is, show that in general

\[
\frac{\partial \ln C}{\partial \ln r_i} = w_i,
\]

where \( w_i = r_i x_i / \sum_{j=1}^2 r_j x_j \) is the \( i \)th factor share. Use this result to derive the conditional factor demand in share form for the first input.

b) State the parameter restrictions you would need to impose to ensure that the symmetry condition holds.

c) Considering your results in part b, state the restrictions on the \( \alpha_i \)'s that would render the translog cost function linear homogeneous in factor prices.
3. Glenwood Grill is a trendy bistro in Raleigh. Most people who dine at Glenwood Grill come to see and be seen. There are, however, 10 hard-core customers per evening who come regularly and don’t care how many other people are present. The number of additional customers who appear depends on how many people they expect to see. In particular, if people expect that the number of customers per evening will be $x$, then the number of people who actually come is $y = 10 + 0.8x$.

a) Solve for the equilibrium nightly attendance.

b) Suppose now that one additional hard-core customer joins the group of regulars. Like the other 10, he eats at Glenwood Grill every night no matter how many other people dine there. Solve for the new equilibrium number of customers.

c) Suppose now that everybody bases expectations about tonight’s attendance on last night’s attendance and that last night’s attendance is public knowledge. Suppose further that on the first night that Glenwood Grill opens, attendance is 20. What will be the attendance on the second night? What is the limiting value that the attendance will tend toward over time?

4. Assume that a firm can hire two types of workers differentiated by their productivity: $k_g$ types with productivity $k = 2$ and $k_b$ types with productivity $k = 1$. The cost of achieving a given level of education is greater for $k_b$ type workers than for $k_g$ type workers. In particular, the cost of $e$ units of education for a $k$-type worker is $c(e, k) = e/k$ and the utility function is given by $U(w, e, k) = w - c(e, k)$, where $w$ represents a wage.

a) Does a worker’s education level influence her productivity? What would be the optimal education level if firms had the same information as workers as to the value of $k$?

b) Now assume that worker productivity is not observable by firms, but that the education level is. Furthermore, assume that firms believe that education greater than or equal to a certain level $e_0$ is a signal of high productivity, while education less than this level signals low productivity. Accordingly, firms offer wages such that $w(e) = 2$ if $e \geq e_0$ and $w(e) = 1$ if $e < e_0$. Given these wages, calculate the level of education that each type of worker will choose.

c) Maintaining the assumptions in b), find the necessary conditions on $e_0$ such that education is an effective signal of productivity.
5. Assume that two factors, wood \((x_L)\) and labor \((x_w)\), are used in fixed proportions by a representative firm in the production of toothpicks \((y)\). Specifically, two units of labor are used in conjunction with one unit of wood to produce a single unit of toothpicks. The inverse demand function for toothpicks is given by \(p = 100y^{-2}\). Suppose also that the representative firm faces an upward sloping (inverse) supply schedule for labor given by \(r_L = 10x_L^{1/2}\). Lastly, initial cost shares of wood and labor are \(w_L = w_w = \frac{1}{2}\). Obtain an estimate of the demand elasticity of wood, assuming that the toothpick industry is competitive.

6. Perdue Farms (the principal) contracts the production (grow-out) of chickens with independent farmers (agents). While tending chickens, a farmer may exert two effort levels: good or bad. If the farmer exerts good effort there is 25% chance that the entire flock will be wiped out by avian influenza (a contagious disease) and if he exerts bad effort there is 75% chance that the entire flock will die. The farmer’s utility is \(U(w,e) = 100 - (10/w) - v\), where \(e\) is effort, \(w\) is the wage paid by the principal to the farmer, and \(v\) takes the value 2 if effort is good and 0 if effort is bad. The farmer’s reservation utility is set at \(U^0 = 0\). If the flock survives, the value of output (broiler meat) is 20; if the flock does not survive, the value of output is zero. The principal’s profit is simply the value of output minus the wage paid to the farmer. The principal is risk neutral.

Calculate the optimal contract and the effort that the principal desires under the following conditions on the farmer’s behavior:

a) symmetric information (effort observable and hence contractible);

b) asymmetric information (effort unobservable hence not contractible).
Macro Prelim - January 2003

Do any five of the following six questions. The questions have equal weight in determining your total score.

1. It is often said that wartime expenditure “stimulates” the economy. Use a Cass growth framework to analyze the effect of wartime expenditure on the economy. For simplicity, assume labor supply is fixed and that population growth and technical progress are zero. Also assume that the only effect of the war on the economy is the expenditure effect; in particular, there is no disruption of economic supplies (such as petroleum), no draft of civilians to serve as soldiers, and so forth. Start in a steady state corresponding to peace, and analyze the effect of a temporary war. Consider separately the cases where the war is

(i) unanticipated up to moment the it starts

(ii) initially is unanticipated but then becomes anticipated some time before it starts

2. Suppose you are trying to explain real money balances per capita across a sample of countries. Among the independent variables you might include in your regression are: real income per capita, a nominal interest rate, the inflation rate and the real wage rate.

(A) Explain why a nominal interest rate would be considered for a regression explaining real money holdings and discuss what sign you would expect the coefficient to have.

(B) Explain why the real wage rate would be considered for a regression explaining real money holdings and discuss what sign you would expect the coefficient to have.

(C) Do you see any problem with including the inflation rate and a nominal interest rate in a regression equation? Discuss.

3. A prominent politician recently remarked in a Wall Street Journal editorial (April 2, 1997) that

“Inflation is caused by too many dollars chasing too few goods, not by too many people working. Inflation results when government prints too much money...Far from being inflationary, real expansion of the economy ‘soaks up’ inflation, creating more goods for the same amount of money to pursue.”

A. Evaluate the assertion that inflation is caused by “too many dollars chasing too few goods.”

B. Evaluate the assertion that “real expansion ‘soaks up’ inflation,” that is, whether real
growth reduces inflation.

C. Discuss the conditions under which an increase in the number of people working "predicts" an increase in the inflation rate.

4. In March 2001 the United States Economy entered a recession according to the NBER Business Cycle Dating Committee.

(A) What is a recession and why are they generally considered to be undesirable?

(B) The current recession is often attributed to a sharp and unexpected drop in investment spending on telecommunications and computer equipment. Evaluate the assertion that an unexpected drop in investment spending can cause a recession.

(C) President Bush has argued that "his" tax cuts in 2001 have helped generate a recovery from the recession and that further tax cuts in 2003 will further aid the recovery. Evaluate the assertion that tax cuts are a potent instrument for ending a recession.

5. In the Solow and Cass growth models, the growth rate is exogenous and therefore unaffected by fiscal policy (taxes, spending). Can fiscal policy affect the economy's growth rate in an endogenous growth framework? Consider the following simple case.

(a) The production function is AK:

\[ Y = AK \]

(b) Utility is constant relative risk aversion:

\[ U(C) = (C^{1-\gamma} - 1)/(1-\gamma) \]

(c) The government collects income taxes

\[ T_y = \alpha Y \]

and consumption taxes

\[ T_c = \beta C \]

where the tax rates \( \alpha \) and \( \beta \) are exogenously set by the government and satisfy

\[ 0 < \alpha, \beta < 1 \]

\[ \alpha + \beta < 1 \]
Government purchases $G$ are set by the government to equal total tax revenue collected and are not productive. The government spends all the revenue it receives on $G$, does not borrow or lend, and does not print money.

(d) Population growth is zero, and labor per person is fixed. Labor therefore is a constant contained in the parameter $A$.

Using the foregoing simple framework,

(A) Show how each of the three elements of government fiscal policy ($\alpha$, $\beta$, $G$) affect the economy's growth rate.

(B) Explain the economic intuition behind your results.

6. Suppose we divide the population into groups.

(A) First, we divide people depending on whether their income increased (group 1) or decreased (group 2) from last year. We then select two people, one from each group, who have the same income this year. On average, how would the level of these two people’s consumption this year compare? Explain.

(B) Next, we discard the grouping in part A and regroup people by whether they work for themselves (self-employed) or for somebody else (employees). It is a fact that the incomes of self-employed people are on average more variable from year to year than are the incomes of employees. On average, how would the marginal propensities to consume compare across the two groups? Explain.

(C) Suppose that, irrespective of which group a person is in, an increase in his permanent income leads an increase in consumption that is much smaller than the increase in permanent income. Briefly explain possible reasons for this observation that are consistent with your answers to parts (A) and (B).
NORTH CAROLINA STATE UNIVERSITY
ECONOMICS CORE PRELIMINARY EXAMINATION

Microeconomics
Tuesday, July 8, 2003

Answer all six questions using the writing pads provided. Write on one side of the paper only and start each question on a new page. Questions are weighted equally.

In questions where the answer depends on particular assumptions, be specific as to the assumptions to make. Your answers should demonstrate your command of economic reasoning.

1. Consider the following distance function for a consumer:

\[ d(q_1, q_2, u) = \alpha(q_1, q_2) + \eta(q_1, q_2)/u, \]

where

\[ \alpha(q_1, q_2) = \gamma_1 q_1 + \gamma_2 q_2, \quad \eta(q_1, q_2) = \beta_1 \ln q_1 + \beta_2 \ln q_2, \]

and where \( \gamma_i, \beta_i \geq 0, i = 1, 2 \) and \( q_i \)'s are quantities consumed.

(a) What additional restriction(s) on the parameters are required in order for \( d(.) \) to obtain the desired homogeneity property?

(b) Explicitly obtain the Antonelli (i.e., compensated) inverse demand for the first good.

(c) Derive the Antonelli matrix \( A \) at the bundle \( (q_1, q_2) = (1,1) \). Verify that terms on the main diagonal of \( A \) meet the required sign restrictions. Is symmetry satisfied?

2. Short answer:

(a) Consider the following normal-form representation of a game with pay-off matrix:

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
</tr>
</thead>
</table>
| \hline
| Player 1 | Left | Right |
| Up       | A,B   | C,D   |
| Down     | E,F   | G,H   |

where the first letter in each cell represents the payoff to Player 1 and the second letter the payoff to Player 2.

(i) If (Up, Left) is a dominant strategy equilibrium, then what inequalities must hold among A,B,C,D,E,F,G,H?

(ii) If (Up, Left) is a Nash equilibrium, then which of the above inequalities must be satisfied?

(iii) If (Up, Left) is a dominant strategy equilibrium, must it be a Nash equilibrium also?
2. Short answer (continued):

(b) A pure monopolist possesses a production technology characterized by the cost function

\[ C(r_1, r_2, y) = (a_i r_i + a_i r_i) y, \quad a_i > 0, \quad i = 1, 2, \]

and faces a demand schedule given by

\[ p = 100y^{-2}. \]

Is there anything wrong here? Briefly explain why or why not.

3. Consider a simple general equilibrium framework with 2 consumers (A and B), two goods (X and Y), one production factor L, and an externality S (smoke) that enters into the production function of each good and adversely affects both consumers. Smoke is considered to be a pure public bad in the sense of non-rivalry and non-excludability. Utility functions are given by \( U_A = U_A(X_A,Y_A,S) \) and \( U_B = U_B(X_B,Y_B,S) \), production functions are given by

\[ X = X(L_x, S_x); \quad Y = Y(L_y, S_y), \]

and adding-up constraints must hold such that \( L = L_x + L_y; \)

\[ X = X_A + X_B; \quad \text{and} \quad Y = Y_A + Y_B. \]

(a) Derive normative conditions for economic efficiency (Pareto optimality) and discuss (interpret) your results.

(b) Derive the analytical expression for the smoke tax that will make the competitive equilibrium Pareto-optimal? (Write the consumers' budget constraint as:

\[ M + \tau(\cdot) = p_x X + p_y Y, \]

where \( M \) represents exogenous income, \( \tau(\cdot) \) is the compensation for smoke damage suffered expressed as a function of the individual's decision variables, \( p_x \) and \( p_y \) are output prices. The price of L is \( p_L. \))

4. A recent proposal regarding alternative fuels in the U.S. argues that consumers of these fuels (e.g., synthetic diesel fuel) should be paid a price subsidy to encourage use. However, the government would also tax away any extra income to consumers generated by the subsidy. Assume that all consumers are identical and have identical preferences given by \( u = u(x, y) \),

where \( x \) is consumption of alternative fuels, \( y \) denotes a composite of all other consumption goods, and \( u(x, y) \) is a strictly quasi-concave function. The representative consumer's budget constraint is \( p_x x + y = m \) where the unit price of other goods has been normalized to unity. Let \( s \) denote the per-unity subsidy rate. Discuss the economic implications of this proposed policy on the representative consumer of alternative fuels. Will she be better off, worse off, or indifferent to this proposal? Would the policy be successful in terms of encouraging alternative fuels consumption? Discuss. (Hint: A properly constructed/interpreted graph may be necessary and is almost surely sufficient.)
5. A used-car sales person and a potential buyer negotiate over the price of an old car. The seller believes that the buyer values the car at $500 with probability $x$ and at $1,000 with probability $(1-x)$. The buyer believes that the car is worth $250 to the salesperson with probability $(1-x)$ and $750 with probability $x$. 

(a) Under what circumstances should trade occur? How does the probability of efficient trade depend on the value of $x$?

(b) If a mediator who wants to maximize the expected gains from trade specifies that the trade should occur at a price of $p$ when the seller claims a value of $250 and the buyer claims a value of $1,000, derive two conditions on $p$ that ensure that both parties will report honestly.

(c) Solve for the value of $x$ that makes both incentive constraints exactly hold as equalities. Note that for lower values of $x$, trade is more likely to be efficient but it cannot always be attained. Explain why.

6. A utility produces electricity to meet the demands of the only city on an island nation. The price it can charge for electricity is fixed by a government commission, and it must meet all demand at that price. Because the climate never varies throughout the year, it turns out that the amount of electricity demand is always the same over every 24-hour period; demand, however, does vary between day (6:00 a.m. to 6:00 p.m.) and night (6:00 p.m. to 6:00 a.m.). During the day $\bar{y}_1$ units of electricity are required, while at night $\bar{y}_2$ are required. Total plant output during any 24-hour period must therefore equal $\bar{y}_1 + \bar{y}_2$. The utility produces electricity according to the production function:

$$y_i = A(KF_i)^i, \quad i = \text{day, night},$$

where $K$ is plant size and $F_i$ is tons of fuel. The firm must build a single plant; and once built it cannot change the size of the plant, nor can it alter the size of the plant between day and night.

(a) If a unit of plant size costs $r_k$ per 24-hour period and a ton of diesel costs $r_f$, what size plant will the firm build?

(b) How much fuel will be required during the day? During the night?
There are six (6) questions on this exam. You must do five (5) of them. Do not do more than five questions. All questions have equal point value.

1. A number of countries are incurring large deficits in their central government budgets; Japan, Germany and the United States are prime examples. For a variety of reasons these governments are reluctant to raise taxes or to reduce expenditures. Consequently, the governments balance their budgets by issuing debt which will require future budget surpluses to pay interest and principle.

   (a) Could these governments finance their deficits with new money creation? Is there any limit to the budget deficit that could be financed by money creation? Discuss.

   (b) Why don't governments choose to finance deficits with money creation? Should they? Discuss.

   (c) How would the economy's real interest rate differ between the cases where the government does and does not finance its deficit with money creation? How do those rates compare to the rate that prevails when the government balances its budget by collecting enough lump-sum taxes to pay for its expenditures? Explain.

2. Suppose you want to study the behavior of nominal interest rates and you have the following model of the economy:

   (1) \( y^* = u_i \)  
   Aggregate supply; \( u_i \) is a serially independent disturbance

   (2) \( y^d = y_0 - y_1 r_t + y_2 y^e_{t+1} + e_t \)  
   Aggregate demand; \( y_t, y_2 > 0; r_t \) is the expected real rate of interest for the current period, \( y^e_{t+1} \) is expected income for the next period and \( e_t \) is a serially independent disturbance

   (3) \( \pi_t = \pi_0 + p \pi_{t-1} + v_t \)  
   Inflation process; \( 0 \leq p < 1 \); \( v_t \) is a serially independent disturbance

   (a) Suppose you wanted to test the Fisher Hypothesis (known as the Fisher Effect) about the behavior of nominal interest rates. What regression would you estimate? How would you interpret the estimated coefficients? If \( p = 0 \) would you be able to test the Fisher Hypothesis? Discuss.

   (b) What determines whether the yield curve (i.e., the plot of the term structure) has a positive or negative slope in this economy? Discuss.
3. Suppose a firm has a production function \( F(K) \), pays \( G \) units of consumption for each new unit of capital \( K \) purchased, faces adjustment costs \( C(I) \) in installing capital with \( dC/dI \) and \( d^2C/dI^2 \) both positive and where \( I \) is investment, and faces a constant exogenous interest rate \( r \) and constant exogenous depreciation rate \( \delta \).

   a. Prove formally that the firm chooses a path of capital that sets the marginal product of capital equal to the user cost of capital:

   \[
   F'_K = (r + \delta)(G + C') - \frac{d\psi}{dt}
   \]

   where \( \psi \) is the costate variable from the current-value Hamiltonian. To carry out your proof, set up and explain the firm's optimization problem; then solve the problem using the maximum principle. Assume there is no uncertainty in the problem.

   b. Explain the economic meaning of the two terms constituting the user cost of capital:

   1. \( (r + \delta)(G + C') \)
   2. \( - \frac{d\psi}{dt} \)

4. Suppose we have labor-augmenting technical progress, so that the production function has the form

   \[
   Y_t = F(K_t, A_tL_t)
   \]

   where

   \[
   A_t = A_0e^{gt}
   \]

   and \( g \) is a positive constant. We suppose \( F \) is homogeneous of degree 1 in \( K \) and \( AL \) (that is, there is constant returns to scale). Labor \( L \) is a fixed number of hours per person, but the labor force grows at the exogenous rate \( n \), so that

   \[
   L_t = L_0e^{nt}
   \]

(A) To keep things simple, use the Solow-Swan model (not Cass) to solve for the steady-state of this model. To do that, derive (don't just assert) the usual fundamental differential equation for the evolution of the capital/efficiency labor ratio and then show what steady states that equation has. Discuss the stability properties of each steady state. Illustrate your answer by drawing the usual kind of diagram showing the intersection of the aggregate saving and augmented depreciation functions.
(B) When the economy is in each of its steady states, what are the growth rates of output $Y$ and output per worker $Y/L$? Explain.

(C) Suppose this economy is Western Europe in 1350 AD, when the Black Death struck. The Black Death was a plague that killed about one-third of Europe’s population. Describe the time path of output per person $Y/L$ in the 50 years before the Black Death and the 50 years after it; explain why it has the shape it does. To simplify the discussion, treat the Black Death as happening in an instant rather than being spread over many years, and suppose that the economy was in steady state when the Black Death struck.

5. Consider the following model:

$$m_t - p_t = a_0 - a_1 (\pi^e_{t-1}) + e_t$$
$$m_t = gt + u_t$$
$$\pi^r_{t+1} = p^r_{t+1} - p_t$$

where

$$m_t = \log \text{ of nominal money supply}$$
$$p_t = \log \text{ of the price level}$$

$e_t$ and $u_t$ are random error terms with $E(e_t | W_{t-1}) = E(u_t | W_{t-1}) = 0$ where $W_{t-1}$ is the information set at time $t-1$ and includes the values of all variables dated $t-1$ and earlier.

$$p^e_{t+1} = E(p_{t+1} | W_t).$$ People know the values of $g$, $a_0$ and $a_1$.

a. Use the method of undetermined coefficients to find a reduced form solution for $p_t$ as a function of $t$, $e_t$ and $u_t$.

b. What is $p^e_{t+1}$ in this model? Give an intuitive explanation for this result.

c. Find $\partial \pi^e_t / \partial e_t$ and give an intuitive explanation for its sign.

6. Consider the following traditional IS-LM model of the economy:

(i) $Y_t = C_t + I_t + G_t$
(ii) $C_t = a_0 + a_1(Y_t - T_t) - a_2 r_t$
(iii) $I_t = b_0 - b_1 r_t$
(iv) $G_t = G^*$
(v) $T_t = d_0 + d_1 Y_t$
(vi) $M_t / P_t = M^* / P_t$
(vii) \( m^d_t = vY_t - gr_t \)

where

\( Y \) = real output  
\( C \) = real consumption  
\( I \) = real investment  
\( G \) = real government purchases  
\( T \) = real tax payments  
\( M^p \) = nominal money stock  
\( m^d \) = real money demand  
\( P \) = price level  
\( r \) = real interest rate  
\( a, b, c, d, v, g \) = constants  
\( M^*, G^* \) = exogenous values

*Explain* what changes you feel are necessary to bring this model into conformity with modern macroeconomic theory and with the existence of a banking system. (Obviously, the model is linear, whereas the real world is non-linear. Ignore that problem in your discussion and treat the model as a linear approximation to the non-linear real world.)
Answer all six questions using the writing pads provided. Write on one side of the paper only and start each question on a new page. Questions are weighted equally.

In questions where the answer depends on particular assumptions, be specific as to the assumptions to make. Your answers should demonstrate your command of economic reasoning.

1. Imagine a situation where an airport runway is located in the vicinity of a residential subdivision. Suppose that the damage caused by noise can be represented by the following marginal damage function $MD = 10 + 0.5Y$ where $Y$ is the number of flights per day landing or taking off. Suppose also that the airport’s profits can be approximated by the following marginal profit (marginal benefit) function $MB = 30 - 0.5Y$.

(a) Calculate the number of flights per day that the profit maximizing airport authority would select while completely ignoring the damages they inflict on the neighboring residents.

(b) Suppose now that negotiation between the airport authority and neighborhood representatives can take place. What is the optimal number of flights per day that they would reach through the negotiation process and what is the corresponding efficient bribe per flight that residents will end up paying?

(c) What is the dollar amount of the net gain (total gain minus total bribe) in the neighborhood’s welfare in the negotiated settlement compared to the maximum number of flights that the airport would like to choose?

2. A consumer’s utility function possesses all of the usual properties (i.e., satisfies all usual regularity conditions). Moreover, the consumer buys only three goods, giving rise to Marshallian demands $x_i = x^*_i\left(p_1, p_2, p_3, m\right), i = 1, 2, 3$, where $p_i$ denotes price of the $i$th good and $m$ income. Corresponding Hicksian demands are, $x_i = x^*_i\left(p_1, p_2, p_3, u\right), i = 1, 2, 3$, where $u$ denotes utility. Some of the following statements are consistent with utility maximization and some are not. If a statement is consistent, explain why. If it is not, also briefly explain why. Note: assume an interior solution.

(a) $\partial x^*_i/\partial p_1 < 0$ and $\partial x^*_i/\partial p_1 < 0$.

(b) $\partial x^*_i/\partial p_1 < 0, \partial x^*_i/\partial p_1 > 0$, and $\partial x^*_i/\partial m > 0$.

(c) $\partial x^*_i/\partial p_1 = \partial x^*_i/\partial p_1$.

(d) $\partial x^*_i/\partial m < 0, \partial x^*_i/\partial m < 0$, and $\partial x^*_i/\partial m < 0$. 
3. The potash industry in Saskatchewan is characterized by the presence of two firms: the government owned firm (GO) and the privately owned firm (PO). The deposits of potash in this Canadian province are estimated to last between 2,000 and 3,000 years, so firms ignore any scarcity considerations when formulating their business strategies. The demand function for potash is given by \( P = 100 - Q \), where \( Q \) denotes the aggregate output of the entire industry, \( Q = q_{GO} + q_{PO} \), and \( P \) is the market price. The unit extraction costs are estimated to be $40 per ton and there are no fixed costs.

(a) Following Cournot duopoly model, suppose that the firms choose their quantities simultaneously. What is the Nash equilibrium?

(b) Following Stackelberg duopoly model, suppose that the government owned firm (GO) is the industry leader and the privately owned firm (PO) is the follower, solve for the equilibrium quantities produced by two firms and the resulting market price.

(c) Contrast the results obtained in a) and b) with the pure monopoly and the perfectly competitive industry outcomes.

4. A colleague comes to your office with an econometric problem. She has data on output price and quantity and input prices and quantities for a collection of firms. She believes these firms behave in a profit maximizing manner, and her advisor has told her to use an indirect profit function of the translog form. That is, to use a profit function of the form:

\[
\ln \pi = \alpha_0 + \alpha_1 \ln p + \frac{1}{2} \alpha_4 \ln p \ln p + \sum_{i=1}^{2} \beta_i \ln r_i + \frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} \gamma_{ij} \ln r_i \ln r_j + \sum_{i=1}^{2} \lambda_i \ln r_i \ln p
\]

where \( \pi \) denotes profit, \( p \) output price, \( r_i \) the \( i \)th input price and \( \ln \) denotes natural logarithm. She is not sure how to proceed. Your mission is to help her.

(a) Verify for your colleague the logarithmic version of Hotelling’s lemma. That is, show that \( \partial \ln \pi / \partial \ln p = w_0 \) and \( \partial \ln \pi / \partial \ln r_i = -w_i \), where \( w_0 = p y / \pi \) and \( w_i = r_i x_i / \pi \).

(b) Show the parameter restrictions that are required for symmetry to hold.

(c) Briefly explain to your colleague why a translog profit function may not be used to test for constant returns to scale.

5. Short answer.

(a) A cost minimizing firm uses only two inputs and, moreover, has isoquants that are concave with respect to the origin. Will the firm ever use both inputs together? Briefly explain.

(b) A consumer purchases only two goods, and these are (as viewed by the consumer) perfect substitutes. Are the consumer’s preferences homothetic? Briefly explain.
5. Short answer (continued):

(c) A firm uses the production function \( Q = \sqrt{KL} \) where \( Q \) is output and \( K \) and \( L \) are quantities of capital and labor. The wage rate for labor is \( w \) and the rental rate for capital is \( r \). What are the long-run equilibrium factor demands for labor and capital as functions of factor prices and output? What is the output price in long-run equilibrium? What is the firm’s long-run equilibrium profit?

6. An electrical utility firm faces a linear demand function \( p = \alpha - \beta y \) and has a cost function of the form \( C(y) = \delta y + F \), where each of \( \alpha, \beta, \delta, \) and \( F > 0, \alpha > \delta, \) and \( (\alpha - \delta)^2 > 4\beta F \). A governmental agency requires your assistance with regards to regulating this monopoly.

(a) Solve for the firm’s optimal output, price, and profits as functions of parameters.

(b) Solve for the deadweight loss associated with the monopoly, and show that it is positive.

(c) The agency is contemplating requiring the firm to set a price that maximizes the sum of consumer and producer surplus, and to serve all customers at that price. What price will the firm charge? Explain to the agency why, in this case, such a policy is unsustainable.
Macro Comprehensive Exam - January 2004

Do any five of the following six questions.

1. Frydman and Rappoport (AER, Sept. 1987) present evidence that, in terms of the effect on real economic activity, there is no significant difference between a change in the nominal money supply that is anticipated and one that is not. More specifically, in an equation like the following they could not reject the hypothesis the $\alpha = \beta$.

$$y_t = y_t^* + \alpha(M_t - E_{t-1}(M_t)) + \beta E_{t-1}(M_t) + u_t$$

where $y$ is a measure of economic activity and $y^*$ is its steady-state value, $M_t$ is the nominal money supply for period $t$ and $E_{t-1}(M_t)$ is its expected value as formed in the previous period. Assume that $t$ indexes short-time periods, for example a calendar quarter.

Discuss the extent to which this result is consistent with the following three types of models:

a. Intertemporal, optimizing, general equilibrium models with perfectly flexible prices and wages.

b. Lucas imperfect information model.

c. New Keynesian wage contracting models where nominal wages are predetermined for two or more periods.

2. It is often said that wartime expenditure “stimulates” the economy (that is, raises output). Use the Cass growth framework to analyze the effect of wartime expenditure on the economy. For simplicity, assume labor supply is fixed and that population growth and technical progress are zero. Start in a steady state corresponding to peace, and analyze the effect of a temporary war financed by a lump-sum tax. Consider separately the cases where the war is

(i) unanticipated up to the moment it starts.

(ii) initially is unanticipated but then becomes anticipated some time before it starts.
3. Suppose Romer's model of learning-by-doing is a good description of the economy's growth behavior. Each firm's output depends on its own capital $K$ and the general level of technology $T$, which is shared by all firms:

$$Y_t = T_t K_t^\beta L_t^{1-\beta}$$

with $T$ is given by:

$$T_t = AK_{at}^\eta$$

where $K_a$ is the average capital stock (aggregate capital divided by the number of firms) and $\eta = 1-\beta$; this is the learning-by-doing part of the model. We suppose that all firms are the same, so that each firm's capital stock $K = K_a$.

Suppose that utility is constant relative risk aversion:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

where $c = C/L$. Suppose also that labor $L$ is fixed and not growing.

(a) Derive the market equilibrium balanced growth rate of this economy.

(b) Derive the socially optimal balanced growth rate.

(c) Explain the economic reason that the two growth rates differ.

4. The conduct of monetary policy is commonly analyzed in terms of a Taylor Rule of the form

$$i_t = \delta + \pi^* + \gamma_\pi(\pi_{t-1} - \pi^*) + \gamma_y(y_{t-1} - y^*_{t-1}),$$

where $i^*$ is the target for the federal funds rate, $\delta$ is the equilibrium real interest rate (presumed to be constant), $\pi^*$ is the desired steady-state inflation rate, $\pi_{t-1}$ is lagged inflation and $(y_{t-1} - y^*_{t-1})$ is the lagged deviation of actual output from equilibrium output. $\gamma_\pi$ and $\gamma_y$ are positive parameters.

If an equation of this form is estimated for U.S. data the estimate of $\gamma_\pi$ varies with the sample period. Before 1979 the estimates are less than 1 and after 1979 the estimates are greater than 1. It so happens that before 1979 U.S. monetary policy was ineffective at controlling inflation and since 1979 it has been quite effective at controlling inflation. Is this coincidence or is there some reason to expect a value of $\gamma_\pi$ greater than 1 would lead to better inflation control than a value less than 1? Discuss.
5. Assume that the representative agent’s utility function is

\[ U_t = \ln c_t + \ln m_t \]

where \( U \) designates utility, \( c \) is consumption and \( m \) is real money balances and \( \ln \) designates natural logarithms. Walsh shows that the first-order condition involving money can be reduced to

\[ \frac{U_m}{U_c} = \frac{i}{1+i} \]

where \( U_m \) and \( U_c \) are partial derivatives and \( i \) is the nominal interest rate on a one-period bond.

(a) How is the steady-state value for real money balances related to the steady-state rate of growth of the nominal money supply? Explain.

(b) Is faster nominal money growth preferred to slower nominal money growth? Explain.

6. Suppose households choose consumption but have fixed labor \( L \), where \( L \) is the same for all households. There are \( H \) households. Utility is logarithmic

\[ U(C_t) = \ln C_t \]

and is the same for all households. All households have infinite horizons and the same rate of time preference \( \rho \). Each household’s current wage \( W_t \) fluctuates randomly from period to period. The distribution function for \( W_t \) is the same for every household: normal with mean \( W \) and variance \( \sigma^2 \). By the law of large numbers, the average wage for the economy is constant at \( W \); households with low wages are offset by households with high wages, leaving the average wage unchanged. Given this restriction, aggregate income is constant at \( HLW \) (number of households times hours per household times wage per hour); equivalently, the sum of household incomes always equals \( HLW \). The economy uses money in the form of currency supplied exogenously by the government. There is no real growth of any kind (no population growth, no technical progress), and the nominal money stock \( M \) is constant.

Suppose that the variance \( \sigma^2 \) of household wage rates was always expected to be constant but then unexpectedly increases to a new value, where it is expected to stay forever. Explain what effect this change would have on

(a) aggregate consumption \( C \),

(b) the price level \( P \),

(c) the real interest rate \( r \).
ECONOMICS CORE PRELIMINARY EXAMINATION  
NORTH CAROLINA STATE UNIVERSITY  

Microeconomics  
Tuesday, June 22, 2004  

Answer all six questions using the writing pads provided. Write on one side of the paper only and start each question on a new page. Questions are weighted equally.

In questions where the answer depends on particular assumptions, be specific as to the assumptions to make. Your answers should demonstrate your command of economic reasoning.

1. Consider a consumer with utility function \( u(x) \), where \( x \) is an \( n \)-vector of goods. Assume the consumer faces price vector \( p \) for the goods, and has income \( m \). The Frisch profit function for these preferences is then defined as

\[
\Pi^F(r,p) = \max_x \{ ru(x) - px \}
\]

where \( r \) is the 'price of utility.'

(a) What overall properties would you ascribe to the Frisch profit function? How would you use envelope theorem results to derive Frisch demand functions \( x_i^F(p,r) \), \( i = 1, \ldots, n \) and the Frisch utility function \( u^F(p,r) \)?

(b) Based on your results in (a), explain how would you obtain the Frisch expenditure function \( e^F(p,r) \). The price of utility \( r \) is, of course, unobservable. Explain how you could use the Frisch expenditure function to obtain \( r \) in terms of observables. Explain how you would derive Marshallin demands \( x_i^m(p,m) \), \( i = 1, 2 \) with this information.

2. Short answer:

(a) You own four fishing boats. You may send all or some portion of your boats to the east or west end of a large lake, equidistant from port. Production at the east end occurs according to \( y = 100x \), where \( y \) is pounds of fish and \( x \) is number of boats. Production at the west end is given by \( y = 140x - 10x^2 \). Fish populations at either end of the lake are completely homogeneous and independent, and current yields are sustainable indefinitely. All costs aside from fuel are fixed. You currently send two boats each to the east and west ends. Should the current allocation of boats be changed to maximize profits? Briefly justify your answer.

(b) Suppose that a consumer has budget constraint \( m = px \) and preferences \( u(x) = x \). Suppose initially that \( p = 2 \) and after a price increase \( p = 4 \). Income remains constant at \$8. What is the compensating variation associated with this price increase?
3. An approach often proposed for dealing with the welfare-distorting effects of a monopoly is to levy some form of tax on the firm. Let the monopolist have a profit function given by

$$\pi = p(y)y - \alpha y$$

where $p$ is price, $p(y)$ is the monopolist's (downward sloping) demand curve, $y$ is output, and $\alpha$ is constant marginal cost. Consider the following:

(a) Assume there is a sales tax that is proportional to the value of sales (total revenue), so that $\pi = p(y)y - \alpha y - \tau p(y)y$, $\tau > 0$ is the profit function. Obtain comparative static results $dg/d\tau$ and $dp/d\tau$. Will imposing such a tax move the monopolist closer to a competitive outcome? Explain/discuss.

(b) What if instead the monopolist’s profits were taxed at a marginal tax rate of $0 < \tau < 1$. How might your answers in part (a) change? Briefly explain.

4. Consider a relationship between a principal and an agent in which only two results, valued at 25,000 and 50,000 are possible. The agent must choose between three possible efforts. The probability of each of the results contingent on the efforts is given in the following table:

<table>
<thead>
<tr>
<th>Efforts</th>
<th>25,000</th>
<th>50,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_1$</td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>$e_2$</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>$e_3$</td>
<td>0.75</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Assume that the principal is risk-neutral and that the agent is risk-averse, with their respective preferences described by the following functions:

$$B(x, w) = x - w \quad \text{and} \quad U(w, e) = \sqrt{w} - v(e)$$

with $v(e_1) = 40, v(e_2) = 20$ and $v(e_3) = 5$. The reservation utility level of the agent is $U_0 = 120$.

(a) Write down the optimal labor contracts under symmetric information for each effort level and the profits obtained by the principal in each case. What effort level does the principal prefer?

(b) Write down the optimal contracts when there is a moral hazard problem. What is the optimal effort level and the contract chosen by the principal? What is the consequence of moral hazard?
5. Consider an economy with two goods, called X and Y, and two people, called "X-lover" and "Y-lover". The X-lover who consumes x units of good X and y units of good Y enjoys utility of $2\ln(x)+\ln(y)$. The Y-lover who consumes x units of X and y units of Y enjoys utility $\ln(x)+2\ln(y)$. Each person is endowed with 3 units of each good.

(a) If the prices of X and Y are each 1 per unit, how many units of each good will be supplied or demanded by each person?

(b) Use your answer to show that there is a competitive equilibrium at which the price of each good is 1.

(c) Show that in this case the competitive equilibrium allocation maximizes the total utility of all the people in the economy. Argue that this implies that the allocation is efficient.

6. Consider a risk-neutral farmer endowed with land that can be either farmed in which case it generates income y or enrolled into the Conservation Reserve Program (CRP). The CRP requires farmers to submit bids, and if the bid is accepted, the farmer receives the rental payment b equal to the value of the submitted bid in exchange for removing the land from agriculture. Therefore, the net benefit of enrolling the land into the CRP is simply b - y.

(a) Let $\beta_i$ be the unknown largest possible bid that farmer i ($i=1,2,\ldots,n$) can submit and still win acceptance into the program and assume that all farmers share common beliefs about $\beta_i$ which is given by a continuously differentiable probability density function with a full support on $[0,\bar{\beta}]$ where $\bar{\beta}$ is the bid cap (i.e. max allowable bid). Hence the probability that a bid is accepted is given by $P(b \leq \beta_i) = 1 - F(b)$ where $F(b) = \int_{0}^{b} f(u)du$ is the cumulative density function of $f$. Solve for the optimal bid $b^*$ that the farmer submits assuming that n is large such that farmers surely take the bids of others as independent of their own. Derive the comparative statics result describing how the optimal bid varies with the agricultural cost (foregone income) of retired land y.

(b) Now suppose that the participants in the CRP are determined by the combination of bids and environmental characteristics of the submitted land as measured by the environmental score s. Farmers are informed of their scores $s_i$ before placing their bids and they condition on $s_i$ accordingly. In particular, you need to assume that $s > s'$ implies that $(Fls)$ first order stochastically dominates $(Fls')$. Solve for the optimal bid under asymmetric beliefs and show how the optimal bid conditional on the environmental score varies with the score.
Macroeconomics Comprehensive Exam - Summer 2004

Do any five of the following six questions. The questions have equal weight in determining your grade.

1. Milton Friedman posited the well known recommendation that monetary policy should create the optimal quantity of money by generating a zero nominal interest rate. Edmund Phelps argued that Friedman's recommendation is sub-optimal because it minimized seigniorage which required an increase in distorting taxes.
   
   a) Explain why a zero nominal interest rate results in an optimal quantity of money.
   
   b) Explain how a central bank would create a zero nominal interest rate.
   
   c) Explain what seigniorage is and why Friedman's recommendation requires sacrificing seigniorage.

2. Consider the Sidrauski Money-in-the-Utility Function model with no depreciation and no population growth. You are given the following information:

   \[ u_t = \ln c_t + \ln m_t \quad \text{and} \quad y_t = f(k_{t-1}) = (k_{t-1})^{0.5} \]

   where \( c \) is per capita consumption, \( m \) is per capita, real money balances and \( k \) is per capita capital.

The representative agent maximizes

\[ \sum_{t=0}^{\infty} \beta^t u_{t+1} \]

by choosing \( c_t, k_t \) and \( m_t \) subject to the constraint that

\[ w_t = y_t + k_{t+1} + \tau_t + m_{t+1}(1 + \pi_t)^{-1} = c_t + m_t + k_t \]

where \( w \) is real wealth, per capita, \( \tau \) is real, per capita, transfers of money to the private sector and \( \pi_t \) is the rate of inflation from period \( t-1 \) to period \( t \). There are no bonds in the economy.

   a) Does the growth rate of the nominal money supply affect the steady-state value of the real money supply in this model? Explain why.

   b) Does the growth rate of the nominal money supply affect the steady-state value of the capital stock in this model? Explain why.

Support your answers to a) and b) by deriving and making reference to the appropriate first order conditions.
3. Suppose Romer's model of learning-by-doing is a good description of the economy's growth behavior. Each firm's output depends on its own capital $K$ and the general level of technology $A$, which is shared by all firms:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

with $A_t$ is given by:

$$A_t = B K_{at}^{1-\alpha}$$

where $K_a$ is the average capital stock (aggregate capital divided by the number of firms); this is the learning-by-doing part of the model. We suppose that all firms are the same, so that each firm's capital stock $K = K_a$.

Suppose that utility is constant relative risk aversion:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

where $c = C/L$. Suppose also that labor $L$ is constant.

Finally, suppose that government collects taxes according to the tax function

$$T_t = \eta + \tau Y_t$$

where $\eta$ and $\tau$ are constants. The government spends all revenue collected on economically worthless projects.

Derive the market solution for the economy's balanced growth rate and explain the economic reason that it depends the way it does on $\eta$ and $\tau$. 
4. Consider a household with the following characteristics:

a. Utility is logarithmic: \( u(c_t) = \ln c_t \).
b. Household income is entirely exogenous; that is, income is a time-varying endowment \( y_t \) completely beyond the household's control.
c. Initial holdings of financial assets are zero: \( A_0 = 0 \).

As always, the household chooses a consumption path to maximize lifetime utility

\[
\sum_{t=0}^{\infty} u(c_t)(1 + \rho)^{-(t+1)}
\]

subject to its lifetime budget constraint

\[
\sum_{t=0}^{\infty} \frac{c_t}{(1 + r)^{t+1}} = \sum_{t=0}^{\infty} \frac{y_t}{(1 + r)^{t+1}}
\]

where \( \rho \) is the rate of time preference and \( r \) is the constant real rate of interest.

(1) Suppose also that income in each period unexpectedly rises by the amount \( \epsilon \); that is, income in each period now is the old value \( y_t \) plus \( \epsilon \):

\[
y_t^{\text{new}} = y_t^{\text{old}} + \epsilon
\]

What is the relation between \( \epsilon \) and the change in \( c_0 \) (consumption in the initial period) if \( r > \rho \)? Derive the answer formally and then explain the economic intuition for your result.

(2) Suppose now that \( r = \rho \). Consider two possible models for the evolution of income:

(A) \( y_t = \bar{y}_t + e_t \)

\[
\bar{y}_t = \bar{y}_{t-1} + u_t \quad \text{[driftless random walk]}
\]

(B) \( y_t = \bar{y}_t \)

\[
\bar{y}_t = \bar{y}_{t-1} + u_t + \theta u_{t-1} \quad \theta > 0 \quad \text{[driftless IMA(1,1) process]}
\]

Explain how the household would adjust its consumption in period \( t \) to an unexpected increase in period \( t \) income in the two cases.
5. Consider two economies - Classico and Keynesiana. In Classico prices are perfectly free to adjust instantly to variations in supply and demand; markets are continuously in equilibrium. In Keynesiana prices adjust slowly to variations to supply and demand. The dynamics of prices in Keynesiana may be due to menu costs (as in Mankiw), multi-period contracts (as in Taylor or Fischer) or gradual updating of information sets (as in Mankiw - Reis). In Keynesiana both prices and inflation are “sticky.” In Keynesiana the quantity of goods produced is determined by the quantity demanded at the “sticky” prices.

In either economy the nominal money supply is determined by a random walk process without any trend, that is, \( M_t = M_{t-1} + m_t \) where \( m_t \) is a random variable that is serially independent. If an econometrician tests the Fisher hypothesis about interest rates and inflation in the two economies, she will get different results for Classico and Keynesiana because the time-series properties of inflation are different in the two economies.

Which economy will provide evidence supporting Fisher’s hypothesis? Why?

6. Suppose individual households have fixed labor \( L \), where \( L \) is the same for all households, but household consumption can vary over time. Utility is logarithmic and the same for all households:

\[
U(c_t) = \ln c_t
\]

All households have infinite horizons and the same rate of time preference \( \rho \). The average real wage \( W \) in the economy is constant, but each household’s real wage \( W_t \) fluctuates randomly about this average. Households with low wages are offset by households with high wages, leaving the average wage unchanged. Given this restriction, aggregate income is constant at \( NWL \), where \( N \) is the number of households; the sum of household incomes always equals NWL. Households use money to buy consumption goods because using money entails lower transactions costs than barter. We can suppose household demand for money is determined by the Baumol-Tobin model, though nothing in this problem depends on that supposition. The only kind of money is currency supplied exogenously by the government. There is no real growth of any kind (no population growth, no technical progress), and the nominal money stock \( M \) is constant. (This is just a Lucas island economy in which people can work for other people instead of just for themselves, which is why there are wage rates, and in which there are no aggregate shocks, only local shocks.)

Suppose that the variance \( \sigma^2 \) of household wage rates was always expected to be constant but then unexpectedly falls to a new value, where it is expected to stay forever. Explain what effect this change would have on the general equilibrium values of

(a) aggregate consumption \( C_t = \sum c_t \)

(b) the price level \( P \)

(c) the real interest rate \( r \)
1. Consumer Theory

An individual has the following utility function: \( u = x - \frac{4}{y} \). The goods consumed are \( x \) and \( y \), and the exogenous prices of those goods are \( p_x \) and \( p_y \) respectively. The individual's income is \( M \). Both \( x \) and \( y \) have to be non-negative, but this is particularly important for \( x \) where the non-negativity restriction is binding for some prices and income. You should consider both cases: Case 1, \( x>0 \) and Case 2, \( x=0 \).

a. Assuming \( x>0 \), derive the uncompensated demands for \( y \) and \( x \). What is unusual about the demand for \( y \)?

b. Use the demand for \( x \) to determine the values of \( p_x \), \( p_y \), and \( M \) for which the non-negativity constraint will be binding.

c. When the non-negativity restriction is binding, what is the demand equation for \( y \)?

d. Derive the indirect utility function for each of the two cases and present them in the simplest possible forms.

e. As a check on your mathematics, use Roy's Identity to derive the uncompensated demand for \( y \) in each of the two cases.

f. Using the indirect utility function, derive the expenditure (or cost) function for this individual in each of the two cases.

g. Derive the compensated demand for \( y \) in each of the two cases. What is unusual about each of these?
h. Now consider the welfare effects of a change in the price of $y$. Suppose $M = 8$, $p_x = 4$, and initially $p_y = 2$. Subsequently, $p_y$ falls to 1. Would $x$ be strictly positive in both these circumstances? Using the demand function(s) for $y$, calculate the change in consumer surplus and the compensating and equivalent variations for this price reduction. Explain the relationship between these three values given this utility function.

i. How would your answers to part h be different if income had been 3? Explain.

2. **Theory of the Firm**

Consider a firm that produces output $Q$ using two inputs, $L$ and $K$. It has only three production processes available to it:

- A: $4L$ and $2K$ produce $1Q$
- B: $2L$ and $4K$ produce $1Q$
- C: $3.5L$ and $3.5K$ produce $1Q$

Each process is infinitely divisible and replicable. They can be combined in fractional amounts.

a. Is the unit isoquant (the isoquant for output equal to one) convex to the origin? Is the input requirement set convex? Discuss the economic implications for the firm's decisions.

b. Derive the firm's cost function and conditional factor demands. (Because of the discrete technologies, you won't want to use standard Lagrangian techniques.)

c. Discuss the difficulties in deriving the unconditional factor demands (profit-maximizing factor demands) for a perfectly competitive firm with this technology.

3. **Externalities**

A chemical manufacturer is located on a small river and discharges some waste into the river. A catfish farmer located downstream is the only person affected by the wastes. The chemical, $x$, is produced at a cost of

$$C = 10 + 2x + \frac{1}{2}x^2$$

and is sold in a competitive industry. The production of a unit of $x$ generates one unit of waste or effluent. The damage, $D$, caused to the catfish farmer by $E$ units of effluent is
\[ D = \frac{1}{2}E^2. \]

Initially, the price of the chemical is 10. The profits of the catfish grower are 50 if there is no effluent. Show your work and explain. (If there is a range of possible outcomes, discuss the bounds.)

a. If the chemical manufacturer ignores the catfish operation, how much of the chemical will be produced and what will be the profits of the two firms?

b. If the chemical manufacturer must compensate the catfish farmer for damages, how much chemical will be produced and what will be the profits of the two firms?

c. If the chemical manufacturer has the right to release the effluent but the farmer pays him to reduce the effluent, how much chemical will be produced and what will be the profits of the two firms?

d. Suppose the price of the chemical falls to 4 and the manufacturer is liable for damages. How much chemical will be produced in the short-run and the long-run? (State any assumptions you make about the fixed costs.)

e. Suppose the price of the chemical falls to 4 and the catfish farmer must induce the manufacturer to reduce the amount of effluent. How much chemical will be produced in the short-run and the long-run?

4. **Oligopoly with Fixed Costs**

Consider a market with two oligopolists who produce an identical product. The aggregate demand for the product as a function of the price, \( p \), is given by

\[ D(p) = \begin{cases} a - bp & \text{for } a \geq bp, \\ 0 & \text{for } a < bp, \end{cases} \]

where the parameters \( a, b > 0 \). For each firm the cost of producing \( y \) units of output is given by the function

\[ C(y) = \begin{cases} F + dy & \text{for } y > 0, \\ 0 & \text{for } y = 0, \end{cases} \]

where the parameters \( F, d > 0 \).

a. Calculate the Cournot-Nash equilibria (with and without production for each firm) as a function of the parameters \( a, b, F \) and \( d \). Your answer should include a
statement of the conditions necessary for the output of each firm to be strictly positive (hint: a firm operates if profit is non-negative). Be sure to analyze the case when the fixed costs are high and the case when they are low.

b. Describe in words what outcomes (in terms of price and output) might occur if the two firms engage in collusion. Be sure to distinguish between the case when monetary transfers between firms are feasible, and the case when they are not.

c. Suppose now that the game between the two firms is repeated either an infinite or a finite number of times. Describe (in words) possible equilibrium outcomes for the two cases. Be sure to state the importance (if any) of the firms’ discount factor.

5. General Equilibrium

Consider a two-person exchange economy with preferences represented by the utility functions

\[ U_A = \ln x_A + 2 \ln y_A, \]
\[ U_B = 2 \ln x_B + \ln y_B, \]

and endowments given by \( \omega_A = \omega_B = (1,1) \). Calculate the Competitive Equilibria (if any) and all Core allocations (if any). Describe the theoretical relationship between Competitive Equilibria and Core allocations, and validate it for this economy.

6. Contract Theory

Consider a principal-agent problem with three exogenous states of nature, \( \theta_1, \theta_2 \) and \( \theta_3 \); two effort levels, \( e_L \) and \( e_H \); and two output levels, distributed as a function of the state of nature and the effort level as follows:

<table>
<thead>
<tr>
<th>State of nature</th>
<th>( \theta_1 )</th>
<th>( \theta_2 )</th>
<th>( \theta_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.25</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Output under ( e_L )</td>
<td>18</td>
<td>18</td>
<td>1</td>
</tr>
<tr>
<td>Output under ( e_H )</td>
<td>18</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The principal is risk-neutral, while the agent has a utility function \( \sqrt{w} \) when receiving a monetary compensation \( w \), minus the cost of effort, which is normalized to 0 for \( e_L \) and to 0.1 for \( e_H \). The agent’s reservation utility is 0.1.

a. Derive the first-best contract when efforts and outputs are observable.
b. Derive the second-best contract when only output levels are observable.
Comprehensive Examination - Macroeconomics
January 2005

Do any five (5) of the following six (6) questions. The questions have equal value.

1. The aggregate production function for the US is fairly well described by the Cobb-Douglas production function:

   \[ Y = AK^\alpha L^{1-\alpha} \]

A macroeconomic fact of the US economy (and possibly others) is that output responds to a temporary productivity shock in a hump-shaped manner. That is, suppose the parameter \( \alpha \) temporarily rises (that is, jumps up, stays there for a while, and then returns to its original value). Then output obviously rises directly because a higher \( \alpha \) gives higher \( Y \). However, output continues to rise after the moment when \( \alpha \) jumps up. Similarly, when \( \alpha \) jumps back down to its original value, output immediately falls but then continues to fall for a while.

(i) Use the Cass growth model to explain this behavior in \( Y \). For simplicity, assume the growth rate of \( L \) is zero. For this part of the question, assume the representative agent believes every change in \( \alpha \) is permanent (that is, he does not understand that they are temporary). Illustrate your answer with a phase diagram.

(ii) Continue with the assumptions and analysis of part (i). Explain the paths of aggregate consumption and labor. Explain also how the paths depend on the wealth and substitution effects of the change in \( \alpha \).

(iii) Repeat part (i), but now assume the representative agent fully understands the temporary nature of the initial change in \( \alpha \). Is it possible for output to start falling before \( \alpha \) drops back to its original value? Explain.

2. The Solow model treats population growth as exogenous. Suppose instead that the percentage population growth rate is proportional to the difference between income per capita \( y \) and some minimal subsistence level of income per capita \( m \); that is,

   \[ (dL/dt)(1/L) = p(y-m) \]

   a. Set up the model with this modification and derive the equation for the growth of the capital/labor ratio.

   b. Draw the usual graph, with appropriate modifications, to show the steady states in the model and discuss their stability.
3. People may be uncertain about their future incomes. Consider a very simple endowment economy in which the household is given a random amount of income each period, with the distribution of income fixed over time and known to the household. The household chooses a path for consumption. The interest rate is constant and equal to the rate of time preference.

   (A) Set up the household optimization problem, stating the objective function and the budget constraint.

   (B) State the Euler condition for this problem.

   (C) Describe the time path of consumption. What does it depend on?

4. Consider the following simple model:

   (1) \((m - p)_t = \pi^*_t + u_t, \alpha < 0\)
   (2) \(m_t = \mu t + e_t\)
   (3) \((m - p)_t = (m - p)_t\)

   where:

   (i) \(m\) and \(p\) are log values of the nominal money supply and price level, respectively
   (ii) \(t\) is time whether as a subscript or as a variable (as in (2))
   (iii) \(u_t\) and \(e_t\) are mean zero, serially independent random variables with finite variance
   (iv) \(\pi^*_t\) is the expected rate of inflation between period \(t\) and period \((t+1)\)

   Expectations are formed as rational expectations. (1) is a money demand function; (2) is a nominal money supply function; (3) is the equilibrium condition.

   (a) Provide a rationale for treating money demand as a function of the inflation rate.
   (b) Find a reduced form solution for the log price level, \(p_t\).
   (c) Find \(\partial(m - p)_t / \partial \mu\) and give an explanation for its sign.
   (d) Find \(\partial \pi^*_t / \partial e_t\) and give an explanation for its sign.

5. There is a common belief that an increase in the nominal money supply or in its growth rate will cause the interest rate to decline. This response is often referred to as the “liquidity effect.” Discuss the characteristics of a model that are necessary to produce this result. Does it matter whether “the interest rate” refers to a nominal or a real interest rate?
6. Milton Friedman has argued that the optimal monetary policy would produce a negative inflation rate that is equal in magnitude to the real interest rate, $\pi = -r$. This recommendation has been criticized because such a monetary policy would require higher taxes to support a given level of government expenditures than would a monetary policy that created positive inflation.

   a) Explain why Friedman’s negative inflation rate would raise welfare.

   b) Explain why a monetary policy that produces positive inflation rather than negative inflation allows for lower taxes for a given rate of government expenditures. Is there a limit to this or can taxes be made arbitrarily low simply by accepting faster inflation?
Macro Preliminary Examination
June 2005

Instructions: Answer 5 of the following 6 questions, starting each question on a new page. Where appropriate, specify a model and derive an answer to the question based on that model. You should support your conclusions by manipulation of the model. (Your analysis may be verbal, graphical, mathematical or some combination of these). Be careful to specify any assumptions you make in conducting your analysis. Unsupported conclusions, even if correct, are not convincing.

1. The U.S. Treasury has announced that it may resume issuing 30-year bonds which were discontinued in the 1990s. Currently the longest term of any bond issued by the Treasury is 10 years. The stated reason for this is that long-term interest rates are currently at historically low levels and the Treasury believes that it can reduce its financing costs by issuing longer term, rather than shorter term, bonds. Based on your understanding of the interest rates on bonds of differing maturities do you think this explanation makes sense? Discuss.

2. Academic economists, in general, earn less than economists who work for investment banks. However, academic economists have much more stable incomes than do investment bank economists. Given this, how do you think the consumption behavior of these two groups compares? Discuss.

3. From The Federal Reserve System’s website (http://www.federalreserve.gov/generalinfo/faq/faqfrs.htm),

“The Federal Reserve's income is derived primarily from the interest on U.S. government securities that it has acquired through open market operations. ... After paying its expenses, the Federal Reserve turns the rest of its earnings over to the U.S. Treasury.”

This suggests that by increasing the rate at which it acquires government securities through open market operations, The Federal Reserve System could contribute more to the U.S. Treasury and reduce the government’s budget difficulties. Discuss any disadvantages and limitations involved in such a plan.

4. Many commentators on the U.S. economy argue that the U.S. national saving rate is too low. How would one decide whether the saving rate is too low or too high? Discuss this issue within the context of the Solow model, the Cass model, and the Diamond overlapping-generations model?
5. A controversy that never seems to end is whether discretion or rules should be used in making monetary policy. Using an appropriate model of your choosing, discuss the benefits and costs of rules and discretion. Include in your answer a discussion of any problems in implementing rules.

6. There is a large literature showing that the introduction of computers into the production process did not have important effects on output until sometime in the 1990s. One possible explanation for this is that in the 1970s and 1980s extensive training was necessary to make the workforce able to use the new technology. By the 1990s, so this story goes, the workforce was sufficiently computer-literate that adding computer-based capital no longer required substantial adjustment costs. Discuss the effects of this reduction in computer-training costs on investment and the capital stock. (Assume that these costs are only associated with adding new capital and that the reduction in these costs is known to be permanent.)
ECONOMICS COMPREHENSIVE EXAMINATION
NORTH CAROLINA STATE UNIVERSITY

Microeconomics
Microeconomics: Tuesday, June 14, 2005, 9am - 1pm

Answer all six questions using the writing pads provided. Write on one side of the paper only and start each question on a new page. Questions are weighted equally.

In questions where the answer depends on particular assumptions, be specific as to the assumptions to make. Show your derivations and explain your logic carefully. Your answers should demonstrate your command of economic reasoning.

1. Tax Incidence

The government of a developing country needs to increase its revenues, and is considering imposing a tax of 1 Shilling on every kilogram of rice sold. A heated debate ensues within the Cabinet over whether or not this would be a sound policy. The Minister of Interior argues against the tax. He claims that a tax on rice would be harmful to consumers, because rice is the dominant staple food consumed daily by nearly every person in the country. The Minister of Agriculture argues against the tax on the basis that it would be harmful to farmers, since rice is the most important agricultural commodity produced in the country. The Minister of Finance argues in favor of the rice tax on the grounds that because both supply and demand of rice are very inelastic, the tax would create relatively less distortions within the economy than taxing any other commodity.

a. If you were charged with helping the President assess the arguments of the various ministers, what would you tell him? Provide an economic analysis of the efficiency and distributional impacts of the proposed tax.

b. Assume that the tax on rice described above is in fact imposed. A new debate ensues within the cabinet as to how the tax should be levied. The Minister of Agriculture argues that the tax should be levied on consumers at the point of purchase. The Minister of Interior argues that it should be levied on producers at the farmgate (when it is sold to marketing agents). What advice would you give to the President on this matter?
2. **Duality in Consumer Theory**

The utility function in this problem is unusual and probably unrealistic. However, it will allow you to demonstrate your understanding of consumer duality relationships. The utility function in not continuously differentiable, so calculus will not be of much help. Nonetheless, it is straight-forward to explicitly write the various dual forms, which you should do. In writing some of the functions, you may need to consider more than one case.

There are two goods, x and y, \((x,y \geq 0)\) in an individual’s utility function.

\[
U(x,y) = \max (x,y),
\]

so the level of utility is equal to either x or y, depending on which is greater. The individual faces exogenous prices, \(p_x\) and \(p_y\), and has exogenous income \(M\).

a. Write the uncompensated demand for \(x\).
b. Write the indirect utility function.
c. Write the expenditure function.
d. Write the compensated demand for \(x\).

e. Now graph the direct utility function in \(x,y\) space, the indirect utility function in \(r_x,r_y\) space where \(r_i = p_i/M\), the expenditure as a function of \(p_x\), and the uncompensated and compensated demands for \(x\). Be sure to label the axes and any relevant points on the axes. Draw the graphs carefully enough to reveal any important relationships.

f. *Extra credit.* If you have drawn them correctly, the indirect utility function should look familiar, but the direct utility function should not. Yet they are dual to each other. Explain.
3. Markets and Trade

Consider a hypothetical yield-enhancing seed variety that lowers the marginal cost of producing wheat in areas in which the temperature never falls below freezing. Because some farms are located in areas where frosts occur, the variety is not universally adoptable. That is, there are effectively two regions of the country, one in which all farmers adopt the new, more profitable wheat variety (with an attendant supply shift), and another in which no adoption occurs. Finally, wheat supply exceeds wheat demand in the frost-free, adopting region, whereas wheat demand exceeds wheat supply in the non-adopting region. For simplicity, assume that there are zero transportation costs.

a. Assume that the economy is closed with respect to wheat so that the wheat price is determined in the domestic market. Describe the welfare effects of adoption of the new variety on the following four types of households:

i. Producers in the adopting region
ii. Consumers in the adopting region
iii. Producers in the non-adopting region
iv. Consumers in the non-adopting region

b. Now assume that the economy is open with respect to wheat, so that the wheat price is determined on the world market. Further assume that the country’s share of world trade in the commodity is negligible, so that the world price of wheat is unaffected by changes in output within the country. Describe the welfare effects of adoption of the new wheat variety in the frost-free region (on the same four types of households).

c. Describe the effects on inter-regional trade between the two regions for both the closed and open economy cases.
4. **Oligopoly and Game Theory**

There are two firms, firm 1 and firm 2, which produce a homogeneous product, $q$. Let production by firm 1 be $q_1$ and production by firm 2 be $q_2$. They have different cost functions: $C_1(q_1) = c_1q_1$ and $C_2(q_2) = c_2q_2$ where $c_1$ and $c_2$ are constants and $c_1 < c_2$. The inverse market demand is

$$p = a - b(q_1 + q_2).$$

**Initially, assume they make their moves simultaneously and follow Cournot behavior.**

a. Set up the profit maximization problem for the first firm and derive the first-order condition. Solve for the reaction function for firm 1.

b. By analogy, what is the reaction function for firm 2?

c. Solve for the equilibrium quantity produced by each firm. Which is larger? What is the market equilibrium quantity if both firms are producing?

d. Graph the reaction functions and equilibrium in $q_1, q_2$ space, labeling all intercepts with their values.

e. Are there any circumstances under which firm 2 will not produce any output but firm 1 will? If so, what are they? Are there any circumstance under which firm 1 will not produce any output but firm 2 will? If so, what are they?

Now assume that instead of a simultaneous game with Cournot behavior, the firms play a two-stage sequential game. The firms follow strategies in quantities as before, but now firm 1 is the leader and makes its quantity decision first. (This is the Stackelberg model.)

f. Discuss how to solve this game.

g. Derive the equilibrium outputs of the two firms. (Assume both find it worthwhile to produce.)

h. Compare the output of firm 1 in the Cournot and the Stackelberg models and discuss the implications.
5. Sharecropping

Sharecropping of agricultural production is a common arrangement around the world. A fairly typical sharecropping contract is one in which a landowner supplies a fixed amount of land ("plot") to a tenant for one season. The tenant farms the land, making all farm management decisions, and in return gives a fraction \( s \) of the harvest to the landowner. Beginning with Alfred Marshall, neoclassical economists have been puzzled over the apparent inefficiency of sharecropping arrangements. Yet they are used in many places throughout the world.

a. By using the maximization process of the tenant farmer, demonstrate through any appropriate means (graphical or mathematical) the nature of this apparent inefficiency? For simplicity, let \( Q \) denote output and \( L \) labor and assume that the production function is of the form \( Q = f(L) \) with \( f_L > 0 > f_{LL} \). The price of output is normalized to 1. Note that the landowner’s share of output is given by \( s \cdot f(L) \) and the tenant’s share is given by \( (1 - s) \cdot f(L) \). Assume also that the tenant can earn a wage \( W \) by working in the competitive labor market elsewhere and that the share \( s \) is exogenously determined. (For parts a, b, and c, assume that there is no uncertainty and that all actions are observable by all parties.)

b. Compare and contrast the efficiency implications of sharecropping with two alternative arrangements:
   i. **Fixed rent** – the tenant pays the landowner a fixed amount of cash up-front for the right to farm the plot for one season. The tenant makes all farm management decisions, and keeps the entire output for herself.
   ii. **Hired labor** – the landowner hires labor at wage \( W \), makes all farm management decisions, and keeps the output for herself.

c. The simple model of the tenant farmer in part a ignores the maximizing behavior of the landowner and the effects of the markets. Suppose the landowner can specify the amount of labor supplied by the tenant (although the tenant can always decide to work elsewhere rather than participating in the contract) and the landowner can also specify the size of the share \( s \). Discuss or show how these considerations would modify the conclusions.

d. In actuality, there is risk in farming (due to, for example, weather). The attitudes toward risk may vary between landowners and farmers. Discuss briefly the risk allocation in the fixed rent, the hired labor, and the sharecropping arrangements. Also, the effort of the farmer may not be observable by the landowner. Discuss briefly the implications of this for the various arrangements.
6. **General Equilibrium and Welfare Theory**

Suppose there is a simple exchange economy with two individuals, A and B, and two goods, x and y. The two individuals have the same utility function:

\[ u_i = x_i y_i \text{ where } i = A, B. \]

Individual A is endowed with 40 units of x and 20 units of y, while individual B is endowed with 20 units of x and 40 units of y.

**For the first parts of this question, assume it is a market economy. Although there are only two individuals, assume they are price-takers and there is no trade out of equilibrium. (In other words, there is a Walrasian auctioneer.)** The price of x is \( p_x \).

a. Normalize the price of y to 1. Why is this possible?
b. Solve for the demands for x and y for each individual.
c. Solve for the aggregate demand for x.
d. Solve for the equilibrium price.
e. Solve for the total quantities demanded by each individual in equilibrium. What are the net quantities demanded by each individual in equilibrium.

**Now assume there is no Walrasian auctioneer and the two individuals are at their endowment point.**

f. Define Pareto Optimality.
g. Set up and discuss (but don’t solve) the objective function, constraints, and non-negativity conditions for the nonlinear programming problem used to solve for Pareto efficiency given the utility functions and endowments above.
h. Given the utility function and endowments above, draw the Edgeworth Box, contract curve, endowment point, and indifference curves through the endowment point. Use that information to describe the points that are Pareto improvements from the endowment point.
i. What is the core of this economy?
j. What would happen to the core if the two individuals and their endowments were replicated? Why?
ECONOMICS COMPREHENSIVE EXAMINATION  
NORTH CAROLINA STATE UNIVERSITY

Microeconomics  
Tuesday, January 3, 2006

Answer all six questions using the writing pads provided. Write on one side of the paper only and start each question on a new page. Questions are weighted equally.

In questions where the answer depends on particular assumptions, be specific as to your assumptions. Show your derivations and explain your logic carefully. Your answers should demonstrate your command of economic reasoning.

1. Three short answer questions:

A. SILICON VALLEY

Silicon Valley is probably the world's leading center of information technology development. The number of firms has increased dramatically over the past 20 years. Yet both land and labor are very expensive in the Silicon Valley. Why, if land and labor are so expensive, does Silicon Valley remain the location of so many information technology firms?

B. DIAMOND-WATER PARADOX

Water is extremely important for sustaining life. Diamonds, on the other hand, are utterly inessential for survival. Why then is water usually very cheap (even free in some places), while diamonds are very expensive?

C. MINIMUM WAGE

An economy uses two types of labor: unskilled workers and skilled workers. Individual firms use some of each type of labor. Initially, wages for the two types of labor are determined in perfectly competitive markets. The government imposes a minimum wage law that is higher than the equilibrium wage for unskilled workers but lower than the equilibrium wage for skilled workers. What will be the effects on wages and employment for unskilled labor? What will be the effects, if any, on wages and employment for skilled labor? What will be the effect on firm profits?
2. **SHORT RUN**

Each firm in a competitive industry producing $x$ has an additive long-run production function, $x = 100L^{1/2} + 100K^{1/2}$, where $L$ is labor and $K$ is capital. However, this question is about the short-run, and each firm has a fixed amount of capital, 25 units. They cannot use more $K$ in the short-run, although it would be possible to not use all of the 25 units if they wished.

a. Write out the short-run production function. Sketch the short-run production function. Would they ever choose not to use all 25 units of $K$? What is unusual about this production function?

b. Write out the function for the marginal product of labor, $MP_L$. Sketch this.

c. Since the production function is additively separable, one could think of the initial units of output being produced by capital until all 25 units are used. After that, additional units of output are produced by labor. This part of the question will be easier to solve (you won’t have to worry about non-negativity restrictions), if you work in terms of $x^L$ where $x^L$ is the difference between total production, $x$, and the initial units produced with capital. However, be sure you remember that the various cost functions, etc. will depend on total production and not just $x^L$.

What is the production function for $x^L$? Solve for the short-run cost function, using $w$ for the wage rate and $v$ for the rental price on each unit of the fixed capital. (Since labor is the only choice variable in the short-run, don’t use a Lagrangian, just solve by substitution.) Sketch the short-run total cost function, $TC$, and the short-run total variable cost function, $TVC$, for some arbitrary parametric values of $w$ and $v$.

d. Solve for the marginal cost function, $MC$. Discuss (but you need not derive) the average variable cost function, $AVC$. Discuss (but you need not derive) the average cost function, $AC$. Sketch these on one graph.

e. Now suppose that the wage rate and the rental rate on capital are each equal to 1 ($w = v = 1$). There are 100 firms in this industry in the short-run. Find the short-run supply function for a typical firm and for the industry. (Assume factor prices are not affected by this industry.)

f. If the market demand function is $Q^D = 1,250,000 - 100,000 P$, what is the equilibrium price and quantity in the industry? How much does a firm produce? What are their profits?
3. **LONG RUN**

Now use the same production function, \( x = 100L^{1/2} + 100K^{1/2} \), but consider the long-run.

a. What is the marginal rate of technical substitution of labor for capital?

b. Solve for the conditional factor demands.

c. Solve for the long-run cost function.

d. Solve for the long-run marginal cost function and the long-run average cost function.

e. Can a competitive equilibrium be sustained in the long-run with this production function? If so, how many firms will there be? What will the price be? (Assume factor prices do not change as the industry expands or contracts.) Does this seem realistic?
4. MONOPOLY PRICING

Electricity in Middleville is provided by Middleville Power and Light (MP&L). Under Middleville's unique regulations, MP&L is the only company allowed to provide electricity and MP&L is allowed to charge different rates to each consumer or firm. The only restriction is that each MP&L customer pays the same rate throughout the day for all uses of electricity. Thus the total expenditure for electricity for each customer is simply the quantity consumed times the price.

a. Consider a typical consumer $i$ with an inverse demand for electricity $p_i = p_i(q_i)$, where $dp_i/dq_i < 0$. Let the cost of providing each unit of electrical services to customer $i$ or any other customer be a constant $c$. Set up and solve the maximization problem for the power company. Show how the price charged each customer is related to $c$. If demand functions vary across consumers, will everyone pay the same price?

b. What factors will the customers being charged the highest rates by MP&L have in common? Give a general answer to this question and then compare the prices charges to the two parties in each of the following cases:
   i. Ms. Jones and Ms. Smith both use electricity for heating, but Ms. Smith also has a wood stove.
   ii. Each month half of the cost of operating Ray's Supercollider consists of electricity bills, whereas electricity represents only 10 percent of the operating costs at Joe Bob's Twin Cinemas.
   iii. Al and Bob both use electricity for lights, but only Bob uses it for heating.
   iv. Acme Co. and Bingo Products both use a lot of electricity in their production. The demand for Acme's product is more inelastic.

c. How will MP&L's pricing strategy change over time? Explain carefully.
5. EXPECTED UTILITY

Consider an individual whose utility is a function of her income $Y$ and is given by $U = \sqrt{Y}$. The individual’s income is uncertain. In any given period her income depends on the realization of one of two different states of nature. In a good year, her income will be $\bar{Y} + \delta$. In a bad year her income will be $\bar{Y} - \delta$. Good and bad years occur with equal frequency; hence her average income will be $\bar{Y}$.

a. Is the individual in question risk averse, risk neutral, or risk loving? How do you know?

b. What is the individual’s expected level of utility? Is the expected utility greater than, less than or the same as the utility of expected income?

c. If the variability of income ($\delta$) increases, but average income stays the same, does the individual’s expected utility increase, decrease or stay the same? Demonstrate this graphically.

d. The government is considering two possible actions. One will have the effect of reducing the variability of income by 50%. The other will transfer $1,000 to the individual. Explain how you would determine which of the two policies would make the individual better off.

e. Now consider two individuals, each having the same utility function as above ($U = \sqrt{Y}$). Individual 1 earns either $\bar{Y}_1 + \delta$ (in a good year) or $\bar{Y}_1 - \delta$ (in a bad year), while individual 2 earns either $\bar{Y}_2 + \delta$ (in a good year) or $\bar{Y}_2 - \delta$ (in a bad year). Again, assume that good and bad years occur with equal frequency. Finally, assume that $\bar{Y}_2 > \bar{Y}_1$.

Is individual 1 more, less or equally risk averse than individual 2? You should discuss your answer in terms of both absolute and relative risk aversion.
6. Two short answer questions

A. USED CARS

Why is it difficult to buy a good quality used car?

B. CROP INSURANCE

Crop insurance programs are notorious for having difficulty staying financially solvent. This is true both for national programs covering farmers spread across large geographical areas, and for local programs that involve farmers located in relatively small geographical areas. In both cases, explain why. In your answer, confine yourself to insurance against yield risk.
Macro Prelim Questions, Winter 2006

Answer any 4 (four) of the following questions; all questions are equally weighted. Do not answer more than 4 (four) questions. In the event that you answer more than 4 (four) questions, only the first four answers will be graded.

1. The shopping-time model seems to be a useful way to capture the role of money as the medium of exchange.
   a. Use the shopping-time model of money to derive a utility function that includes money in the utility function.
   b. Is an increase in the real wage likely to increase or decrease the demand for money in the shopping-time model? Explain why.

2. Most discussions of the Solow growth model assume that the rate of population growth is constant and exogenous. Suppose, however, that the rate of population growth increases with income per worker. How would this affect the steady-state path of the economy? To answer this compare two possible behaviors of the population growth rate, \( n \).
   Case 1: \( n = \bar{n} \).
   Case 2: \( n = \bar{n} \) for \( y \leq \bar{y} \) and \( n = \bar{n} + n(y) \) for \( y > \bar{y} \), where \( n(\bar{y}) = 0 \), \( n' > 0 \), \( y \) is income per worker and \( \bar{y} \) is a very low subsistence level of income per worker.
   How do the steady-state conditions of these two cases compare? Explain.

3. The U.S. currently has a large federal government budget deficit and a large trade deficit. Under what assumptions would these two deficits be connected and under what assumptions would they be unrelated? Make sure you explain how the assumptions lead to the conclusions.

4. Virtually all central banks conduct monetary policy by specifying a target value for a short-term interest rate and intervening in financial markets to keep that interest rate close to the target.
   a. Explain why central banks choose a target value for an interest rate rather than choosing a target value for the quantity of reserves, or some other monetary aggregate. That is, why do central banks use an interest rate as their policy instrument rather than some measure of the quantity of reserves?
   b. Discuss the factors that cause a central bank to choose to raise or lower its interest rate target.

5. Discuss the policy options available when a country is hit by an adverse supply shock such as a large increase in world oil prices. Which option would you choose? Defend your choice.
6. Suppose a firm has a production function $F(K)$, pays $G$ units of consumption for each new unit of capital, $K$, purchased and faces adjustment costs $C(I)$ in installing capital where $C' > 0$ and $C'' > 0$ and $I$ is investment. The firm faces a constant exogenous interest rate, $r$, and constant exogenous depreciation rate, $\delta$.

a. Prove formally that the firm chooses a path of capital that sets the marginal product of capital equal to the user cost of capital:

$$F_k = (r + \delta)(G + C') - \frac{d\psi}{dt}$$

where $\psi$ is the costate variable from the current-value Hamiltonian. To carry out your proof, set up and explain the firm's optimization problem and solve the problem using the maximum principle. Assume there is no uncertainty in the problem.

b. Explain the economic meaning of the two terms constituting the user cost of capital:

1. $(r + \delta)(G + C')$

2. $-\frac{d\psi}{dt}$
ECONOMICS COMPREHENSIVE EXAMINATION
NORTH CAROLINA STATE UNIVERSITY

Microeconomics
Monday, June 12, 2006

Answer all six questions using the writing pads provided. Write on one side of the paper only and start each question on a new page. Questions are weighted equally.

In questions where the answer depends on particular assumptions, be specific as to your assumptions. Show your derivations and explain your logic carefully. Your answers should demonstrate your command of economic reasoning.

1. CONSUMER THEORY

Suppose an individual has the following expenditure function, $E(p,u) = k(u)g(p)$ where $p$ is a vector of prices for the $n$ goods, $u$ is a scalar measure of utility, $k(u)$ is a positive monotonic function of $u$, and $g: \mathbb{R}_+^n \rightarrow \mathbb{R}_+^n$.

a. Use Roy’s Identity to derive the uncompensated demand for the $j$-th good, $x_j(p,M)$.

b. Use Shephard’s Lemma to derive the uncompensated demand for $x_j$. (Use $x_j^H(p,u)$ for the compensated demand if you use that in your derivation.)

c. Discuss the Engel curve for good $x_j$.

d. Derive and discuss the income elasticity for $x_j$.

e. Would you expect this expenditure function to be useful empirically? Why or why not?

Now suppose that the expenditure function were slightly more complex, $E(p,u) = k(u)g(p) + f(p)$.

f. Derive the uncompensated demand for $x_j$ by one of the two methods above.

g. Discuss the Engel curve for good $x_j$ now.

h. Derive and discuss the income elasticity for $x_j$ now.

i. Why is this expenditure function of some theoretical interest for aggregate demands? Would you expect this expenditure function to be useful empirically? Why or why not?
2. **JOINT PRODUCTION OF GRAIN AND STRAW**

Consider a farmer who engages in only one productive activity, growing wheat on his farm. Each wheat plant produces two consumable items – wheat and straw. The farmer consumes wheat grain. The farmer’s pet goat eats wheat straw. The farmer is unusually fond of his goat, as is indicated by his utility function:

\[
U(C_G, C_S) = \sqrt{C_G \cdot C_S}
\]

where \( C_G \) and \( C_S \) represent the amounts of grain and straw consumed by the farmer and his goat, respectively.

The farmer plants wheat on 10 acres of land, and can choose between planting two varieties of wheat. Each variety produces grain and straw that are identical in terms of nutritional quality and taste. Variety 1 yields 500 kg of grain and 200 kg of straw per acre planted. Variety 2 yields 150 kg of grain and 400 kg of straw per acre planted. He can plant all his land entirely in one variety or the other, or he can allocate a fraction of his land to Variety 1 and the rest to Variety 2.

a. Graphically show the transformation curve for grain and straw. What is the marginal rate of transformation between grain and straw?

b. Assume that the market prices of grain (\( P_G \)) and straw (\( P_S \)) are $0.30 per kg and $0.15 per kg, respectively. How many acres will be planted in Variety 1 and how many in Variety 2? Will the farmer purchase or sell grain? Will the farmer purchase or sell straw?

[HINT: Augment the graph in part (a)]

c. How much grain will the farmer consume? How much straw will his goat consume?

Now assume that the cost of marketing straw is prohibitively expensive, so that the goat can only eat straw that the farmer has produced.

d. Is the farmer’s well-being greater with or without a market for straw? How can you tell?

e. How many acres will be planted in Variety 1 and how many in Variety 2.

f. How much grain and straw will be grown? How does these amounts compare with the earlier situation (when both grain and straw could be traded)? How much grain and straw will be consumed, and how do these amounts compare to with the earlier situation?
3. COST CURVES AND THEORY OF THE FIRM

Suppose you own an airline, Anonymous Air (abbreviated AA) that flies only one route, which is between Smallville and Metropolis. All your customers live in Smallville and want to make daily round-trips to Metropolis. (No one has ever wanted to make the round trip in the other direction.)

It is the short-run. You have a maximum of three planes your airline can use. Each plane can make only one round-trip per day. You pay a rental price of \( r \) per day per plane. Each plane is identical and holds up to 100 passengers, but of course they may not always be full. Each passenger pays the same price, \( p \), for a round trip. Whether the plane is full or not, the Federal Aviation Authority requires that you have five employees on the flight. This cost is \( v \) per flight per day. In addition, you have other costs, \( w \), that vary linearly with the number of passengers, \( q \).

This question should be answered verbally and graphically. For some parts of the questions below, there may be multiple local optima, and you may have to consider both marginal and total conditions in your discussions of the decision process.

For the purposes of this question, let us agree on some terms to be used. A cost that does not vary even if you shut down is a **sunk cost**. A cost that does not vary with the number of passengers on a plane but can be avoided if you eliminate the flight is a **fixed economic cost**. A cost that varies with the number of passengers on a plane is a **variable cost**. **Economic costs** are the sum of fixed economic costs and variable costs.

A. First assume that your lease on the planes requires you to pay for all three whether you use them or not and you are not allowed to sub-lease the planes to other firms. Also the flight crews have a strong union, so you have to pay them whether they fly or not. The only cost that varies with the number of passengers is \( w \cdot q \).

1. Draw and discuss the short-run cost curves for your firm (total in one diagram and marginal/average in another).

2. If there are other airlines that fly between Smallville and Metropolis so that it is a perfectly competitive industry, how would you decide how many planes to fly, how many passengers to carry, and at what fare? Would you ever fly three planes? Why? Would you ever fly zero planes? Why? Would you ever fly one or two planes? Why?

3. If you are the only airline allowed to fly between Smallville and Metropolis so that you have a monopoly, how would you decide how many planes to fly, how many passengers to carry, and at what fare? Would you ever fly three planes? Why? Would you ever fly zero planes? Why? Would you ever fly one or two planes? Why?

(continued on next page)
B. Now assume that you that you are successful in negotiating the right to sub-lease to other firms any planes you do not use. You also negotiate with the union so that you don’t have to pay crews who do not fly. However, the FAA still requires that if you fly a plane it has to have a crew of five.

Now answer the questions in parts 1, 2, and 3 above under these conditions and discuss the differences from your answers to part A.
4. **RESIDENTIAL IMPACT FEES**

Typically, new residential development increases demands for expensive public services like schools, roads, and police protection. Many communities have begun charging “impact fees” to cover these costs. Impact fees are fixed dollar amounts levied on real estate developers for each new home constructed. Both home buyers and real estate developers tend to complain bitterly about impact fees. New homeowners complain that the great bulk of these fees are passed through to them in the form of higher prices. Real estate developers complain that the impact fees reduce demand for new housing and hence they lower industry profits.

Consider a hypothetical community that is considering a $5,000 impact fee on each new house constructed. Assume that there are two kinds of houses, “low-cost” and “high-cost.” Community demand and supply curves for the two types of new houses are as follows:

\[
Q_{LowCost}^D = 400 - 0.002P \\
Q_{HighCost}^D = 300 - 0.0004P \\
Q_{LowCost}^S = 25 + 0.0005P \\
Q_{HighCost}^S = 25 + 0.00015P
\]

a. Are new homeowners correct in stating that the great bulk of the fee will be passed on to them by the real estate developers?

b. Is there a difference between prospective buyers of high-cost houses and low-cost houses in terms of the share of the fee they will pay? If so, which would pay more.

c. Will the imposition of the impact fee reduce the amount of new houses constructed? If so, are there likely to be fewer high-cost houses or fewer low-cost houses built?

An alternative to impact fees would be to impose an ad valorem sales tax on new houses sold in the community. Now consider the impact of a 1.5% sales tax on new houses.

d. Which party – buyers or sellers – will pay a greater share of the increased sales tax?

e. Will the increased sales tax reduce the amount of new houses constructed? If so, how many fewer houses of each type will be built? How do these effects on new house construction compare with the effects associated with the $5,000 impact fee?

Another alternative is to raise property taxes on all properties, both new and old.

f. Briefly discuss the effects of this alternative on the developers and purchasers of new properties and the owners of existing properties.
5. INSURANCE

Blue-eyed people are more likely to lose their expensive watches than are brown-eyed people. Specifically, there is an 80 percent probability that a blue-eyed individual will lose a $1,000 watch during a year, but only a 20 percent probability that a brown-eyed person will. Blue-eyed and brown-eyed people are equally represented in the population.

a. If an insurance company just takes the population average as the probability that someone will lose a watch, what will the actuarially fair premium for watch-loss insurance be?

b. If blue-eyed and brown-eyed people have identical logarithmic utility-of-wealth functions \( u = \ln(W) \) and each has a current wealth of $10,000, will these individuals buy watch insurance at the premium calculated in part (a)?

c. Given your results from part (b), will the insurance premiums be correctly computed? If everyone who buys insurance pays the same premium, what will the actuarially fair premium be? What will the utility for each type of person be?

d. Suppose that an insurance company charged different actuarially fair premiums for blue-eyed and brown-eyed people. How would these individuals' maximum utilities compare to those computed when there was a single premium as calculated in (c)?

e. Suppose that an insurance company was charging the different premiums for people with different color eyes by looking at their eyes. Now someone discovers a way to make contact lenses for the eyes that make it appear that a blue-eyed person had brown eyes. She offers these lenses for sale at a fixed price. Will anyone buy the contacts? Why or why not? Will this affect the equilibrium in the insurance market? If so, how?
6. SHORT ANSWER QUESTIONS

a. In India pairs of oxen are used for plowing fields. It is almost never the case that pairs of oxen are rented alone. Instead, the owner of the oxen is also hired to do the plowing. Moreover, the owner of the oxen is paid based on how many acres he and his oxen plow (rather than on a per day basis). Provide economic explanations for both of these observations.

b. Payday lenders and pawn shops have very similar clientele - people with poor credit. Payday lenders provide workers with a fraction of their take-home pay prior to payday. In exchange, the worker signs over the entire amount of his or her next paycheck to the lender on payday. Pawn shops, on the other hand, loan money to individuals who must provide some valuable item (e.g., jewelry or a musical instrument) for the pawn shop to hold. If the individual does not repay the money that the pawnshop loaned him or her within the specified time period, that person forfeits the valuable item. Otherwise, he or she can get the item back by repaying the amount of money that was advanced to him, plus a small service fee.

Payday lenders charge high fees for their services, while pawn shops charge lower fees. Why?

c. Aunt Tilly is getting older and wants to make out her will to dispose of her estate on her death. The only living relatives she has are a niece, Ann, and a nephew, Bill. She has great affection for one of them and dislikes the other one. However, she is afraid that if she leaves one of them completely out of the will, that person will challenge the will and tie up the estate in court.

To avoid this she decides to leave the $100,000 to the two of them collectively and let them divide it up between themselves under the following rules. When Tilly dies, Ann will propose a split for the estate. If Bill accepts it, they get the money at that time. If he does not, they have to wait one year. At that time Bill proposes a split. If Ann accepts it, they get the money at that time. If she does not, they wait one more year. At that time Ann gets to propose another split. If Bill accepts it, they get the money then. If he doesn’t, neither of them gets anything and the money goes to a charity.

Ann and Bill are fully rational, each wants to maximize their own wealth, each has a discount rate (or rate of time preference) of 5% per year. All three of them (Ann, Bill, and Tilly) are aware of these facts.

Who does Aunt Tilly like best? How do you know? (Your answer should outline the optimal strategies Ann and Bill will follow.)
Macro Prelim Questions, Summer 2006
Final Version, June 13, 2006

Answer any 4 (four) of the following questions; all questions are equally weighted. Do not answer more than 4 (four) questions. In the event that you answer more than 4 (four) questions, only the first four answers will be graded.
1. Consider the Ramsey-Cass growth model. Households maximize the present value of utility where

\[ U = \int_{t=0}^{\infty} e^{-\rho t} u(C(t)) dt, \quad \rho > 0 \quad \text{and} \]

\[ u(C(t)) = \frac{C(t)^{1-\theta}}{1-\theta}, \quad \theta > 0 \].

C is consumption per worker.

A large number of identical firms produce with the production function, \( Y(t) = F(K(t), L(t)) \). K is the capital stock and L is the labor force. We assume the labor force, L, is constant. We assume the rate of technological progress is zero. Further, we assume that all markets are perfectly competitive and that there is no uncertainty.

a) This economy has a government that buys goods at the rate \( G(t) \) and uses those goods in some manner that does not generate utility. The goods are paid for with lump-sum taxes in each period (that is, there is always a balanced budget). Suppose at time \( t^* \) the government announces that it will temporarily increase \( G(t) \) and will keep spending at the increased level until time \( t^* + k \). Describe the dynamic path of the economy in response to this change and discuss how the path will depend on the size of \( k \) (that is, on whether the increased spending is to be maintained for a long or a short period).

b) In part a) it was assumed that government spending was paid for from tax revenue equal to spending in each period. How would your analysis of the increased spending in part a) be different if the increased spending were paid for by borrowing rather than by increased spending? Explain.
2. In an important 1978 paper, Robert Hall argued that consumption spending should evolve as a random walk. In particular, Hall argued that information available in period \( t \) could not be used to forecast the change in consumption from \( t \) to \( t+1 \).

a) Derive this result or discuss the derivation if you cannot reproduce it in detail.

b) Explain why Hall's result is an implication of the permanent income hypothesis.
3. Consider an economy in which
(i) Expenditures are described by an expectations augmented IS curve (current spending depends on expected future income and the expected real interest rate)

(ii) Prices are determined, as in the Mankiw - Reis paper, by

\[ p_t = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t+j} (p_{t+j} + \alpha y_{t+j}) \]

which implies that inflation evolves as

\[ \pi_t = \left( \frac{\alpha \lambda}{1 - \lambda} \right) y_t + \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t+1-j} (\pi_{t+j} + \alpha \Delta y_t) \]

The variables have their standard definitions and should be considered to be deviations from the steady-state path. \( \lambda \) is the fraction of firms that re-optimize their prices each period.

(iii) The Fisher Equation holds in this economy. The economy is populated by rational, utility maximizing households and profit maximizing firms that operate in imperfectly competitive industries. Firms set prices as described by Mankiw and Reis and produce the quantity of goods and services demanded at those prices. There is no growth.

(iv) Monetary policy is determined by a single central banker who chooses an inflation target and conducts open market operations to keep the inflation rate equal to the target, on average. At infrequent random intervals the central banker retires and is replaced by a new central banker. Upon appointment the new central banker chooses a new value for the inflation target and the inflation rate stays constant at the new level until he or she retires. The process of retirement, appointment and change to the new inflation target occurs instantaneously and is a complete surprise when it happens. Economic agents immediately learn the new level of the inflation target and know that another change will not occur in the near future.

a) Suppose the old central banker retires and is replaced by a new central banker who chooses a higher inflation target. Discuss the adjustment process for this economy for the important real and nominal variables including production, real interest rate and nominal interest rate. Explain your reasoning.

b) How would your analysis in a) be different if wages and prices were perfectly flexible and firms operated in perfectly competitive markets with no costs of information (that is, neoclassical determination of prices)? Explain your reasoning.
4. The government budget constraint, in real terms, can be written as
\[ \frac{G_t}{P_t} + \frac{i_{t+1}}{P_t} B_{t+1} = \frac{T_t}{P_t} + \frac{(B_t - B_{t+1})}{P_t} + \frac{(M_t - M_{t+1})}{P_t}, \]
where \( t \) indexes time and
\( G \) is nominal, government purchases of goods and services,
\( P \) is the aggregate price level,
\( i \) is the nominal interest rate,
\( B \) is the number of outstanding, one-period, government bonds with $1 nominal value,
\( T \) is nominal tax receipts, and
\( M \) is the nominal money supply (assume all money is high-powered money).

The same government budget constraint can also be written as
\[ \frac{G_t}{P_t} + (1 + i_{t+1}) B_{t+1} = \frac{T_t}{P_t} + B_t / P_t + \frac{(M_t - M_{t+1})}{P_t}. \]

A staff economist at the central bank proposes that monetary policy can relax the government budget constraint so that government expenditures can be increased without increasing taxes. This economist proposes that the growth rate of the nominal money supply should be increased. The economist’s argument is that faster money growth will raise the right-hand side of the constraint because the term \( (M_t - M_{t+1})/P_t \) will increase in the future. At the same time, the economist argues, the left-hand side of the constraint will be decreased in the future because the term \( (1 + i_{t+1}) B_{t+1}/P_t \) will fall as faster money growth leads to higher prices.

a. Evaluate the first part of this staff economist’s argument in the context of the steady-state. Will a more rapid rate of growth of the nominal money supply increase the future value of \( (M_t - M_{t+1})/P_t \)? That is, holding other factors constant, can a government increase the resources it acquires through increasing the nominal money supply by raising the rate of growth of the nominal money supply? Discuss.

b. Evaluate the second part of this staff economist’s argument in the context of the steady-state. Will faster nominal money growth cause the term \( (1 + i_{t+1}) B_{t+1}/P_t \) to fall in the future? Discuss.
5. Consider the following model:

\[
\begin{align*}
    m_t - p_t &= a_0 - a_1 (\pi^e_{t-1}) + e_t \\
    m_t &= gt + u_t \\
    \pi^e_{t+1} &= p^e_{t+1} - p_t
\end{align*}
\]

where

- \( m_t \) = log of nominal money supply
- \( t \) = the time period
- \( p_t \) = log of the price level
- \( e_t \) and \( u_t \) are random error terms with \( E(e_t | I_{t-1}) = E(u_t | I_{t-1}) = 0 \) where \( I_{t-1} \) is the information set at time \( t-1 \) and includes the values of all variables dated \( t-1 \) and earlier.
- \( p^e_{t+1} = E(p_{t+1} | I_t) \). People know the values of \( g, a_0 \) and \( a_1 \).

a. Use the method of undetermined coefficients to find a reduced form solution for \( p_t \) as a function of \( t, e_t \), and \( u_t \). Give an intuitive explanation for the effects of \( t, e_t \), and \( u_t \) on \( p_t \).

b. What is \( p^e_{t+1} \) in this model? Give an intuitive explanation for this result.

c. Find \( \partial(m_t - p_t) / \partial e_t \) and give an intuitive explanation for its sign.
Answer all six of the questions that follow. The questions carry equal weight. Write your answers in the test booklets provided. Answer each question completely and concisely. You have four hours to complete this examination. Turn in this document along with your examination.
1. The U.S. government has, for many years, provided financial support to U.S. agricultural producers. Much of this support has been given through the provision of price supports, which maintain the minimum price that a grower will receive for their output. Assume that the U.S. government decides that the current (free-market) price is “too low” and institutes a price floor (support price) of $P^S$. If market prices are beneath $P^S$, taxpayers make a payment equal to the difference between the support price and the market price. Using standard partial-equilibrium methods of analysis, answer the following questions associated with this program.

(a) First, provide a simple illustration (supply and demand) of the economic effects of this program. Be sure to identify the welfare effects of the program on taxpayers, producers, and consumers. Discuss who wins and loses as a result of this policy.

(b) If the U.S. is a large exporter of the commodity in question (meaning that the actions of the U.S. are sufficient to impact world prices), what happens in international markets when the U.S. institutes this program? Again, be sure to identify the welfare effects on foreign consumers and producers.

(c) Describe what happens (in terms of welfare effects) if technological progress results in productivity gains in the production of this commodity in the U.S.

(d) Suppose now that the U.S. government, alarmed by increasing costs of the program, decides to institute a mandatory production diversion (meaning that producers must agree to cut back on production). A producer is free to leave the program and receive no support (i.e., they will face the free-market price). However, if they stay in the program and continue to receive the higher support price, they must agree to idle a proportion of their total production $\beta$ (where $0 < \beta < 1$). Assume that all producers are identical, such that if one decides to participate, all will participate. Describe the economic (welfare) effects of this program. If the support price is above the market price, will producers always choose to participate in the price support program? Explain why or why not.
2. Consider an individual that must decide how to allocate their limited amount of time. They choose a level of leisure $L$ and consumption of a market good $Q$ to maximize a utility function $U(L, Q)$. The price of the market good is $P_Q$. The individual happens to have two income-earning opportunities—(1) working at home, producing a good $H$ that they subsequently sell at a price of $P_H$ in the market, and (2) working for hire outside of the home. The home-produced good $H$ is produced using only labor $L_H$ according to a production function $H = f(L_H)$, where $f(\cdot)$ displays diminishing marginal productivity with respect to labor. If the individual chooses to work in the marketplace, they earn a fixed wage of $w$. They have a fixed number of hours per week $T$ and must decide how to allocate $T$ among leisure ($L$), work at home ($L_H$), and work in the marketplace ($L_M$).

(a) Set up the individual's utility-maximizing problem. For this part of the question, you can assume that the individual always chooses to work some at home and some in the market, and always consumes a positive amount of leisure (i.e., you may assume interior solutions).

(b) Derive the first-order conditions for utility maximization and provide an economic interpretation for them. Use a diagram to illustrate the solution to the problem.

(c) How would the problem and solution change if you accounted for the fact that $L$, $L_H$, and $L_M$ cannot take on negative values? That is, how would you impose nonnegativity constraints and what would the first-order conditions look like?

(d) What happens to the solution if the individual is able to hire outside labor for their "at-home" job at a much lower wage rate than they themselves face in the market?

(e) What happens if a proportional income tax of $\tau\%$ of total wages is applied only to work outside of the home?

(f) What happens to the allocation of time if hours spent working in home production $L_M$ directly provide utility (i.e., $L_M$ becomes an argument in the utility function)?
3. During the “Green Revolution” in India, new varieties of wheat (which happened to have shorter stalks and more grain) significantly boosted the output of wheat. The “Revolution” can be interpreted as an exogenous shock that raised the value of the marginal product of labor ($VMP_L$) (and land, capital, materials, etc.). Consider a single village that enjoyed the benefits of this technological change (i.e., only this village experiences the revolution). Focusing on the effects on labor in this particular village, answer the following questions.

(a) If the number of laborers in a village remains the same, what did the “Green Revolution” do to the wage and to the village output? Be sure to include a diagram in your answer (labor quantity vs. wage and $VMP_L$). You should identify $VMP_L$ and the areas corresponding to the increase in output, assuming that the quantity of labor is fixed.

(b) If the village faces a perfectly elastic supply of labor at the old equilibrium wage and labor flows into the village in response to an initial rise in wages, what is the whole additional output in the village? How much is earned by labor on the one hand and by all other factors on the other?

(c) How is output elsewhere in the overall Indian economy affected by the inflow of labor into the village? What, therefore, is the net social gain from the “Green Revolution” for this individual village and from allowing labor to flow into the village in response to it? Who gets all the gain—labor or the other inelastically supplied (immobile) factors, such as land?
4. Equilibria in Games

a. The matrix below represents a simple zero-sum game. The game is played once. Players choose among all possible strategies to maximize expected value. The row player chooses R1 or R2 and the column player chooses C1 or C2. The column player pays the row player the amounts indicated when the corresponding pair RX and CX is chosen. What is the Nash equilibrium in this game? (State the players’ strategies and the expected value of the game for each of the players.)

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b. Suppose there are two identical firms in an industry. Marginal cost is zero. The inverse demand function for the industry is \( P = a - bQ \), where \( Q = q_1 + q_2 \). Consider the usual Cournot game in which players chose quantity. What are the Nash (Cournot) equilibrium prices and quantities?

c. Now consider the following entry game. Each of two airlines consider entry into a route. The route that row’s could enter connects with the route that column could enter, and vice versa. Firms have complete information. Either player can take an action at any time; that is, simultaneity is not forced. Entries in the matrix are payoff in the form (Row payoff, Column payoff)

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What is (are) the Nash equilibrium (equilibria) in this game?

d. Compare these three equilibria. In what sense are they equilibria? Do they make sense as equilibria? That is to say, are they robust against the actions of thoughtful players? Could they be used to predict behavior?
5. Competition

a. The widget industry is a constant cost industry composed of identical firms. All firms are price takers. Entry is unrestricted and new entrants can duplicate the production capabilities of existing firms. Demand facing the industry is \( Q = 300,000 - 1000P \), where \( Q \) is monthly demand. The cost function for any firm in this industry is \( C = 200,000 + 0.05q^2 \), where \( q \) is the firm’s output in any month. Solve for the long-run equilibrium price, the output for the industry and the output of the individual firm.

b. For the equilibrium you determined in the question above, what is the short run (i.e., no entry) supply function?

c. Suppose that we are in the equilibrium presented in part a., and the government suddenly requires that all widget manufacturers put in a radiation hazard detection system at their loading docks. The firms will rent these systems for $1000 per month and incur no other costs from having these systems in the plant. How will this requirement affect price, industry output, and the output of an individual firm in the short run? (The short run is defined as involving no entry or exit) How will it affect price, industry output and the output of an individual firm in the long run? You do not have to compute an answer, but rather must state the direction of any changes.

d. Again, start from the equilibrium in part a. Suppose manufacturing a widget uses up one pound of aluminum and the price of aluminum has just gone up by 14%. State the short run and long run changes, as in part c, of the increase in the price of aluminum.

6. Concentrated markets

Microsoft is currently introducing Windows Vista, its new operating system for business and home computing. As you almost certainly know, the Microsoft Windows is the operating system that operates on most desktop and laptop computers. Windows Vista allegedly will have better security, better screen appearance, and perhaps somewhat better facilities for file management than the old operating system, among other improvements. However it will serve the same basic purposes as the old operating system.

Suppose you have been hired to advise Microsoft regarding their pricing of this new product. What are the principal influences on the market for this product? What is (or are) Microsoft’s main competition for this new operating system? What other factors will they wish to take into account in pricing this product? Comment generally on what pricing strategies you would recommend for this new product?
1. Assume that the central bank seeks to minimize the expected value of the following loss function:

\[ L_t = (y_t - y^* - k)^2 + \pi_t^2 \]

The economy has a Lucas-type supply function:

\[ y_t = y^* + \beta(\pi_t - \pi_0^e) + u_t \]

The error term, \( u_t \), has expected value zero, constant variance (\( \sigma_u^2 \)), and is serially uncorrelated. Everyone knows the parameters of both functions and the values of \( y^* \) and \( k \) and that \( k > 0 \).

Suppose that the central bank is able to control the inflation rate, \( \pi_t \), and, therefore, treats \( \pi_t \) as a policy variable. That is, the central bank can choose \( \pi_t \) each period. The central bank can observe \( u_t \) prior to choosing \( \pi_t \), but the public cannot observe \( u_t \) before forming \( \pi_0^e \).

a. What value of \( \pi_t \) would the central bank choose if it takes \( \pi_0^e \) as given?

b. Is an announced policy of \( \pi_t = 0 \) credible? Explain your answer in terms of the expected values of the loss function.

c. Suppose one could legislate a policy rule of \( \pi_t = 0 \), which people believed would be followed. Would it dominate allowing the central bank to use its discretion if the public had the same loss function as the central bank and formed its expectations rationally? Discuss your answer?

2. Some models, for example the Sidrauski money-in-the-utility-function model, conclude that “Money is superneutral.”

a. What does it mean to assert that “Money is superneutral”?

b. Outline a formal model in which money is not superneutral and discuss why it is not superneutral in this model.
3. Dynamic inefficiency is an important issue in growth theory.

   a. What is meant by dynamic inefficiency?


   c. Is it optimal to eliminate dynamic inefficiency? Discuss.

4. The federal government is considering switching from an income tax to a consumption tax, i.e. the tax base would be a person’s consumption rather than income. Discuss the likely macroeconomic effects of such a change.

5. Consider the Real Business Cycle approach to macroeconomic fluctuations.

   a. Discuss the basic model.

   b. What empirical implications does the model have for the cyclical behavior of real wages, the price level, and unemployment?

   c. Are these implications supported by the U.S. experience?

6. The short-run versus long-run distinction is important in macroeconomics. For example, undergraduate texts often present a short-run and a long-run aggregate supply curve. Discussions of monetary policy often refer to short-run and long-run Phillips curves. Indeed, monetary policy would be a pretty boring topic if it were not for the hypothesis that monetary policy actions can have short-run effects on economic activity (aggregate production, employment, etc.).

   a. Outline a formal model that rationalizes a short-run response in economic activity to monetary policy actions that differs from the long-run response.

   b. Discuss the empirical support for the model you present.
1. Consider a model economy where output of a firm depends on aggregate amounts of capital and labor in addition to the factors of production employed by each firm. That is,

\[ Y_{it} = A(K_{it}^\theta L_{it}^{-\beta})K_{it}^\alpha L_{it}^{1-\alpha} \text{ where } \alpha > 0, \beta > 0. \]

\( Y_{it} \) is output produced by the representative firm, \( K_{it} \) and \( L_{it} \) are the amount of capital and labor the firm uses in production, and \( K_i \) and \( L_i \) are their aggregate counterparts. The competitive firm maximizes profits: \( \Pi_{it} = Y_{it} - (r_t + \delta)K_{it} - w_tL_{it} \), where \( r \) is the rental rate of capital, \( \delta \) is the depreciation rate and \( w \) is the wage rate.

As usual, the individuals maximize utility:

\[ \int_0^{\infty} \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \]

subject to

\[ \dot{k}_t = w_t + r_k + \eta_0 - c_t \]

where \( y_t, c_t, k_t \) are output, consumption and capital per capita, respectively, and \( \eta_0 \) is the initial wealth. Assume for simplicity that there is no population growth.

a) Solve for \( r \) and \( w \) in terms of \( k \) using the firm’s first order conditions and the fact that each firm will choose the same capital per labor ratio in equilibrium.

b) Set up the Hamiltonian and obtain the first order conditions of the individual’s maximization problem, and the dynamic equation for consumption in the competitive economy.

c) Under what special condition will this model exhibit endogeneous growth? Explain the rationale behind that condition. What does the growth rate depend on?
2. Assume that the infinitely lived representative individual maximizes

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) + \theta \log(1 - L_t) \right] \right] \]

where \( \beta \) is the discount factor. The total amount of time available to agents in each period is normalized to 1. Thus, \( 1 - L_t \) is the leisure in period \( t \). The individual supplies labor to the firms in return for the wage rate \( w_t \), accumulates capital and rents it out to the firms at rate \( R_t \). Hence, the individual's budget constraint is given by the following:

\[ C_t + K_{t+1} = R_t K_t + w_t L_t \]

The production function of the perfectly competitive firm is:

\[ Y_t = (A_t L_t)^\alpha K_t^{1-\alpha} \]

where \( A_t \) is the productivity shock. Assume for simplicity that there is no depreciation, and the firm maximizes profits: \( \Pi_t = Y_t - R_t K_t - w_t L_t \)

There is no growth in the steady state, therefore, technology, consumption, capital and output are constant in the steady state. Assume that \( A = 1 \) in the steady state.

a) Find the Euler equation for consumption and interpret it.

b) Find the labor supply equation and interpret it.

c) Find the steady state ratios for \( \frac{L}{K} \), \( \frac{Y}{K} \) and \( \frac{C}{K} \) in terms of the parameters of the model.

d) The solution to this model can be obtained by solving the system of linear expectation equations in \( C, K, L \) and \( A \). Obtain the system of equations. In obtaining the system, use the following assumptions:

- the percentage deviations of \( A_t \) from its steady state value follow an AR(1) process: \( \dot{A}_t = \phi \hat{A}_{t-1} + \epsilon_t, E_{t-1}(\epsilon_t) = 0 \)
- consumption and the rental rate of capital are jointly log-normal and homoskedastic. (Hint: if \( X \) and \( Y \) are jointly log normal, then \( \log [E_t(X_{t+1}Y_{t+1})] = E_t[\log(X_{t+1}Y_{t+1})] + \frac{1}{2} \text{var}_t[\log(X_{t+1}Y_{t+1})] \).

3. Assume the following model:

\[ y_t^a = \alpha(p_t - p_t^*) + \gamma y_{t-1} + \epsilon_t \quad 0 < \gamma < 1; \alpha > 0 \quad \text{aggregate supply} \quad (1) \]

\[ y_t^d = \beta(m_t - p_t^*) + \alpha_t \quad \beta > 0 \quad \text{aggregate demand} \quad (2) \]

\[ m_t = \delta m_{t-1} + v_t \quad 0 < \delta < 1 \quad \text{money supply} \quad (3) \]
a) Discuss a theoretical rationale for equation (1), carefully explaining what would determine the value of $\alpha$.

b) Assume that the errors have zero means and constant variances $(\sigma^2_{x}, \sigma^2_{a}, \sigma^2_{t})$, and that they are not serially correlated or correlated with each other. Assume that the public’s expectation, $pte$, is formed rationally but before observing any of the error terms. What are the equilibrium values of $p_t$ and $y_t$? Discuss the intuition behind your answer.

c) Suppose that the money supply equation is changed to $m_t = \delta m_{t-1} + \varphi a_t + \nu_t$
and that the central bank can observe $a_t$ but not $\epsilon_t$ or $\nu_t$ while the public cannot observe any of the errors. What value of $\varphi$ should the central bank pick to minimize the variance of $y_t$? Explain your answer.

4. Consider an economy at its steady state growth rate of real output with a growth rate of the money supply of 3%. Suppose the money supply growth rate were to increase to 6%. Discuss conditions under which the steady state capital-labor ratio would be lower after the increase in the growth rate of money, under which the ratio would be higher, and under which the ratio would be unaffected.

5. The government of country X is running a budget deficit. It decides to raise the needed revenue by printing money at a faster rate. Use a model to explain the conditions under which this plan will work.

6. Some countries set one goal for their central bank: keep inflation at some low target such as 2%. Why would a country want to constrain its central bank in this fashion? Specify a model that would imply that such behavior is rational and give the intuition behind your argument.
Economics Core Preliminary Examination  
North Carolina State University  

Microeconomics  
Thursday, June 14, 2007

Answer all six questions using the writing pads provided. Write on only one side of the paper and start each answer on a new page. Questions all carry the same weight.

In cases where your answer depends on particular assumptions, be specific as to the assumptions you are making. Your answers should demonstrate your command of economic reasoning.

I. True, False, or Uncertain (Explain)

1. The price elasticity of demand (in absolute value) for a contingent input demand (demand holding output constant) is smaller than the elasticity of demand for the input when output price is held constant.

2. A farmer who sells potatoes is observed to increase his consumption of potatoes when price increases. This proves that potatoes are an inferior good.

3. In housing booms, monthly mortgage payments for new homes tend to rise relative to monthly rental rates.

4. A lump sum tax on a competitive firm will lead to no change in output.

5. Unskilled workers receive relatively low wages because their productivity is low.

6. Profits of gasoline stations fall when crude oil prices rise.

II. Consumer Demand

The PIGLOG expenditure function has the general form

$$\ln C(u, P) = a(P) + ub(P).$$

where $C(u, P)$ is the expenditure function for utility $u$ and price vector $P$. The functions $a(P)$ and $b(P)$ are twice continuously differentiable functions. Assume for the sake of convenience $b(P) = \beta_0 \prod_{j=1}^{n} p_j^{\beta_j}$, where $\sum \beta_j = 0$. 


a. Show that the Marshallian consumer demand functions are:

\[ W_i = \frac{\partial a(P)}{\partial \ln P_i} + \beta_i [\ln I - a(P)] \quad \forall i \]

where \( W_i = \frac{PY_i}{I} \) is the expenditure share of the ith good, and \( I = C(u, P) \) is total consumer expenditure.

b. What restrictions must hold on these demand functions in order for them to be derived from an expenditure function?

c. For these demand functions, derive the expenditure (income) elasticities.

d. Suppose you were interested in testing to see if the income elasticity for food would decline as income increases. Is this an appropriate functional form for testing the hypothesis? Explain.

III. Contracts

Consider a contract stipulates that one party, a seller, agrees to provide a widget to another party, a buyer, for an agreed upon sum of money, \( P \). As the parties enter into the contract, the value of the widget to the buyer is \( B \), the seller’s cost of producing the widget is \( C \).

a. Does the contract increase total utility of the parties involved? How do you know?

b. A contractual breach occurs when one party to a contract declines to perform as stipulated in the contract. The non-performing party then is liable for an amount that can be specified in the contract and which sometimes is resolved with recourse to the courts. Ordinarily the breaching party is required to “make the other party whole,” which is to say, pay damages. Suppose the buyer in our example pays \( P \) to the seller upon entering into the contract, and subsequent to that, the seller elects to breach the contract. Suppose further that the court awards the buyer an amount of money equal to \( B \), which remains the buyer’s valuation of the widget. An alternative for the court is to require the seller to provide the widget, what is called specific performance. Does the court’s decision to award compensation increase or decrease utility as compared to the alternative of specific performance? Explain.
c. Sometimes parties to a contract include clauses that specify the payments that must be paid in the event of a breach rather than allowing determination of a party’s actual losses after the fact. For example, our hypothetical widget contract could have specified that in the event that the seller breaches the contract, the seller will pay the buyer some amount B’, which may or may not equal B. Clauses specify a B’ that is quite large relative to B are termed penalty clauses. For B’ clauses generally (i.e. not just penalty clauses) why would the parties to a contract agree to specify a B’?

d. In common-law countries, courts will often refuse to enforce penalty clauses. In light of your answers to the above questions, what economic justification is there, if any, for the common law disposition against enforcing penalty clauses? And in light of that, is there any economic justification for enforcing penalty clauses?

IV. Durable goods

a. Assume that there is fixed demand for some durable good. For now, assume that the durable good does not depreciate. Finally, assume that there is a fixed upward sloping competitive supply of new production of the durable good in each period, with a strictly positive vertical intercept. (You could think of housing, with new construction in any period and a stock of existing houses.) Thus the stock of the durable good unfolds as $S_t = S_{t-1} + f_{t-1}$. Assume that the durable goods of all vintages are perfect substitutes. Characterize the long run equilibrium price and quantity of the durable good. (You can offer this as a general characterization, or if you must, you can assume specific forms for the demand for the durable good and the supply of new production.)

b. Start with the conditions in part a., but now assume that there is a depreciation rate $\delta$. Characterize the long run equilibrium.

c. Again start with the conditions in part a., but now assume that production of the durable good is monopolized. What will change?

V. Regulation and Interrelated Factor Markets

A livestock producer has the option of selling on market A (the spot market) or market B (contracted livestock). The two markets are close, but not perfect, substitutes for livestock used in production of meat. The government decides to restrict the amount sold on market B below its current level.

a. Formulate the profit maximization problem facing a given producer who incurs fixed per unit costs, $c_A$, when selling on market A and fixed per unit
costs, $c_B$, when selling on market B. The cost function of producing livestock is $c = c(Y_A, Y_B)$, where $Y_A$ and $Y_B$ represent the quantities sold on markets A and B, respectively. Derive the first-order conditions (f.o.c.) for profit maximization and provide an economic interpretation.

b. Suppose now the producer faces a restriction of $Y_B \leq \bar{Y}_B$ on the number of livestock that he can sell on market B. How are the f.o.c. for profit maximization affected? Give an economic interpretation. If the restriction is binding, how will the producer’s decision on how much to sell on each market be affected?

c. Starting at initial equilibrium in both markets A and B, suppose now that a restriction is placed on the quantity sold on market B below its initial equilibrium level. What would you expect to happen to prices and quantities of livestock sold on markets A and B as all producers respond to the regulation?

d. What are the welfare effects on sellers of livestock on each market?

VI. Efficiency

Economists regard efficiency as a goal or objective; perhaps the only goal or perhaps one among many. What are the efficiency concepts that economists use? What are the merits of these norms as measures of economic “goodness”? What are the deficiencies of these norms and how serious are they?
INSTRUCTIONS
Answer all parts of all six questions. Questions are weighted equally. Partial credit is awarded and may be important.

1. Short Answer Questions: (Keep these short or you may not have time to finish the exam. Check to make sure you have answered all six parts of this question.)

   a. The income elasticity of demand for food is .3 and the budget share of food is .3. This must mean the income elasticity for non-food is equal to 1.3.

   b. If the number of firms in an industry is small, then the industry is necessarily oligopolistic with each firm producing where $P > MC$.

   c. We have a law of demand, but no law of supply. Nevertheless, we very nearly treat upward sloping short-run supply as a law. Why? And why is long-run supply different?

   d. A tax on gasoline that is returned to consumers in a lump sum subsidy will cause gasoline consumption to fall.

   e. We would expect demand for a specific type of wine like Merlot to be more price elastic than demand for wine as a whole.

   f. Any point on the production possibilities frontier represents an efficient allocation of resources.
2. A competitive firm faces a constraint on how much capital it can purchase: 
\[ \bar{k} - k \geq 0. \]

a. Set up the Lagrange function and find the first-order conditions for constrained profit maximization for labor and capital assuming the constraint is binding.

b. Interpret the first-order conditions. How would you expect the employment of capital and labor to be affected relative to when the firm does not face a constraint on purchasing capital?

c. With the constraint binding, what would you expect to happen to the amount of output produced by the firm? Explain.

3. Let \( P_1 \) be the price of premium wines and \( P_2 \) (\( P_2 < P_1 \)) be the price of lower-quality wines. Both wines are taxed at the same rate \( t \) so that prices of the two wines can be written as \( P_1 + t \) and \( P_2 + t \). Suppose because of news in the medical community about the health benefits of wine, the federal government decides to reduce the tax rate on wine to encourage more wine consumption. How would you expect the tax to affect the average quality of wine consumed? To simplify the analysis, assume that the demand functions for wine are Hicksian demand functions and depend only upon the prices of the two qualities of wine. Finally, assume that the health benefits of wine do not depend on the quality of the wine.

4. Let the utility function be \( U = x^{0.5} + y^{0.5} \)

a. Derive the Expenditure function for the consumer.

b. Derive the Hicksian demand functions for \( x \) and \( y \).

c. Show that the Hicksian demand functions are homogenous of degree zero in prices, negative with respect to own-price changes, and symmetric with respect to cross-price changes.
5. Property. Economists increasingly point to economic institutions in as an important influence on the economic conditions of a country. Among those institutions is a legal system that provides for secure private property rights.
   a. What are property rights?
   b. In what respect is a system of private property presumed in standard economic analysis?
   c. What economic problems are addressed constructively by private property? That is, what problems does property solve in comparison with common ownership and open access to resources? In answering this question, be as comprehensive as you can.

6. Invention. (Parts a. and b. of this question can be answered briefly, and should be. Those two parts will carry less weight than parts c. and d.)

   In a famous paper, Kenneth Arrow stated that “[W]e expect a free enterprise economy will under-invest in invention and research (as compared with an ideal)...”
   a. What are the reasons that we would expect this underinvestment?
   b. A patent system is a remedy, to some extent, to the underinvestment that Arrow expects. A patent is a time-limited exclusive right to practice a qualifying invention. How does a patent remedy this problem? Would you expect a patent to permit full appropriation of the value of an invention? Less than full appropriation? More than full appropriation? Explain. What tradeoff or compromise does a patent entail?
   c. Assume the cost of developing any given patentable innovation decreases over time as widely-held background knowledge and technology advances. Assume further that a patent does allow the inventor to capture the full value of her invention, and a patent is awarded to the first person to invent; that is, the first person to formulate a feasible approach to a particular innovation. No other “inventor” can be awarded a patent on a particular invention. What conclusions can you draw about the economic desirability or efficiency of such institutional arrangements under the assumed conditions.
d. A recent U.S. Federal Trade Commission report notes the problem of "blocking patents," a circumstance that arises when one party holds a patent for a product and a different party holds a patent on an essential improvement. For example, suppose one party has patented the basic architecture for the light bulb, but the innovation has little practical value without using a filament design that is patented by someone else. The claim is that the existence of separate patents will deprive society of the benefits of the innovation, as each patent blocks the other. As an economist, you should have something to say about this problem. Say it. (Well, actually, write it down.)
1. Consider a model economy where output of a firm depends on aggregate amount of capital in addition to the factors of production employed by each firm. That is,

\[ Y_{it} = AK_{it}^\alpha L_{it}^{1-\alpha} K_t^\lambda \text{ where } 0<\alpha < 1 \]

\( Y_{it} \) is output produced by the representative firm, \( K_{it} \) and \( L_{it} \) are the amount of capital and labor the firm uses in production, and \( K_t \) is the aggregate capital stock. The competitive firm maximizes profits: \( \Pi_{it} = Y_{it} - (r_t + \delta)K_{it} - w_tL_{it} \), where \( r \) is the rental rate of capital, \( \delta \) is the depreciation rate and \( w \) is the wage rate.

The individuals maximize utility:

\[
\int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt
\]

subject to

\[
\dot{k}_t = w_t + r k_t + \eta_0 - c_t
\]

where \( y_t, c_t, k_t \) are output, consumption and capital per capita, respectively, and \( \eta_0 \) is the initial wealth. Assume for simplicity that there is no population growth.

a) Solve for \( r \) and \( w \) in terms of \( k \) using the firm’s first order conditions and the fact that each firm will choose the same capital per labor ratio in equilibrium.

b) Set up the Hamiltonian and obtain the first order conditions of the individual’s maximization problem, and the dynamic equation for consumption in the competitive economy.

c) Under what special condition will this model exhibit endogenous growth? Explain the rationale behind that condition. What does the growth rate depend on?

d) Is the competitive equilibrium Pareto-optimal? If not, propose a tax/subsidy scheme that will make the competitive equilibrium Pareto-optimal.
2. Assume that the infinitely lived representative individual maximizes

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) + \theta \frac{(1-L_t)^{1-\gamma}}{1-\gamma} \right] \right]$$

where $\beta$ is the discount factor. The total amount of time available to agents in each period is normalized to 1. Thus, $1 - L_t$ is the leisure in period $t$, and $\gamma$ is the inverse of the elasticity of labor. The utility maximization is done subject to the capital accumulation equation:

$$K_{t+1} = (1 - \delta)K_t + Y_t - C_t - G_t$$

where $G_t$ is the government spending that is financed by lump-sum taxes. The production function takes the following form:

$$Y_t = (AL_t)\alpha K_t^{1-\alpha}$$

where $A$ is the constant level of technology. The government runs a balanced budget every period, and the government spending is subject to shocks. There is no growth in the steady state, therefore, technology, consumption, capital and output are constant in the steady state. Assume that $A = 1$ in the steady state, and the government spending to output ratio is $g < 1$.

a) Find the Euler equation for consumption and interpret it.

b) Find the labor supply equation and interpret it.

c) Find the steady state ratios for $\frac{L}{K}$, $\frac{Y}{K}$, $\frac{G}{K}$ and $\frac{C}{K}$ in terms of the parameters of the model.

d) The solution to this model can be obtained by solving the system of linear expectation equations in $C, K, L$ and $G$. Obtain the system of equations. In obtaining the system, assume that the percentage deviations of $G_t$ from its steady state value follow an AR(1) process: $\hat{G}_t = \phi \hat{G}_{t-1} + \epsilon_t$, $E_{t-1}(\epsilon_t) = 0$.

e) What is the necessary condition for the determinacy of this model?
3. There is a common belief that an unexpected increase in the nominal money supply or in its growth rate will cause “the interest rate” to decline. This response is often referred to as the liquidity effect."

   a. Discuss the characteristics of a model that are necessary to produce this result for the real interest rate.

   b. If a positive shock to the nominal money supply growth rate decreases the real interest rate, can we assume that the nominal interest rate will also decrease? Explain.
4. Monetary policy is conducted by setting a nominal interest rate target and using open market operations to keep the interest rate equal to its target.

a) Explain why monetary policy is conducted this way rather than by setting a target for the nominal money supply and using open market operations to keep the nominal money supply equal to its target.

b) Suppose that the nominal interest rate is equal to its target, but the nominal money supply is larger than it was expected to be when the interest rate target was set. That is, 
\[ M_t > E_t(M_t|_{i_t = i^*}). \]
Is this information relevant to revising the interest rate target? Explain.
5. Consider the following simple model:

(1) \((m - p)_t^d = \alpha \pi^e_t + u_t, \quad \alpha < 0\)

(2) \(m_t = \mu t + e_t\)

(3) \((m - p)_t^d = (m - p)_t\)

where:

(i) \(m\) and \(p\) are log values of the nominal money supply and price level, respectively.

(ii) \(t\) is time whether as a subscript or as a variable (as in (2)).

(iii) \(u_t\) and \(e_t\) are mean zero, serially independent random variables with finite variance.

(iv) \(\pi^e_t\) is the expected rate of inflation between period \(t\) and period \((t+1)\);

\[\pi^e_t = E_t(p_{t+1}) - p_t.\]

Expectations are formed as rational expectations. (1) is a money demand function; (2) is a nominal money supply function; (3) is the equilibrium condition.

(a) Find a reduced form solution for the log price level, \(p_t\).

(b) Find \(\partial (m - p)_t / \partial \mu\) and give an explanation for its sign.

(c) Find \(\partial \pi^e_t / \partial e_t\) and give an explanation for its sign.

\[ u_t = \ln c_t + \ln m_t \quad \text{and} \quad y_t = f(k_{t1}) \]

where \( c \) is per capita consumption, \( m \) is per capita, real money balances, \( k \) is per capita capital and \( f(k_{t1}) \) has a positive first derivative and a negative second derivative.

The representative agent maximizes

\[ \sum_{i=0}^{\infty} \beta^i u_{t+i} \]

by choosing \( c_t, k_t \) and \( m_t \) subject to the constraint that

\[ w_t = y_t + k_{t-1} + \tau_t + m_{t-1}(1 + \pi_t)^{-1} = c_t + m_t + k_t \]

where \( w \) is real wealth, per capita, \( \tau \) is real, per capita, transfers of money to the private sector and \( \pi_t \) is the rate of inflation from period \( t-1 \) to period \( t \). There are no bonds in the economy.

a) Does the growth rate of the nominal money supply affect the steady-state value of the real money supply in this model?

b) Does the growth rate of the nominal money supply affect the steady-state value of the capital stock in this model?

Support your answers to a) and b) by deriving and making reference to the appropriate first order conditions.
Micro Preliminary Exam
Summer 2008

Answer all questions and be sure to explain your answers and show all relevant work. Begin each question on a separate page of paper.

1. Short Questions—Answer (True, False, or Uncertain and explain your answer) Question will be graded on the quality of your explanation.

(a) True, False, or Uncertain. If good $X$ and $Y$ are perfect substitutes, and if both goods are bought and sold in a competitive market, then they will sell for the same price.

(b) True, False, or Uncertain. If lower gasoline prices cause people to drive more miles such that their total gasoline expenditure is unchanged, then there is no benefit to lower gasoline prices.

(c) True, False, or Uncertain. Assume that the supply price of raising corn increases as more is produced. Suppose the husks (leaves) are used to produce biofuel and that the ears (the corn without the leaves) is used for food. If the production of biofuels from corn is stopped (say, due to a better alternative becoming available), then the price of the ears of corn will go up.

(d) True, False, or Uncertain. A $10 per unit tax on a monopoly can cause the price to go up more than $10.

(e) True, False, or Uncertain. Roy is consuming 10 units of $X$ per period. He observes that a $1$ increase in the price of $X$ would be much worse for him than a $10$ decrease in income. If this is a very small change in the price of $X$, then Roy is not maximizing his utility.

(f) True, False, or Uncertain. If income rises in proportion with prices such that the consumer can buy what they were originally consuming, then people will not be better off.
2. You have been assigned to develop contract terms and rates for fire insurance, which will be sold to risk averse homeowners. As a commercial entity, the insurance company is risk neutral. For the first part of this question, assume that any fire will result in a total loss of $L$, where $L$ is the total value of the house (i.e., that there is no partial loss). Define the probability of a fire occurring as $\pi$. The insurance premium (the cost of the protection) is $p$. The insurance policy calls for deductible of $k$, meaning that only a portion $1 - k$ (where $0 < k < 1$) of the losses $L$ will be paid in the event of a fire (meaning that homeowners must pay a fraction $k$ of the losses in the event of a fire).

(a) For a given level of deductible $k$, what is the actuarially-fair premium (i.e., the value of $p$ that will result in zero profits for the insurance company)?

(b) If homeowners are allowed to choose the level of deductible $k$, what level will they select with actuarially-fair premiums in order to maximize expected utility? Assume that the utility of wealth is state-independent (i.e., that $\$1$ from an indemnity payment is valued the same as $\$1$ worth of house). Justify your answer.

(c) Suppose now that the homeowner can “be careful” and thereby reduce the chances of a fire occurring. The level of precautionary activity is $x$, which is supplied at a cost of $C(x)$ where $C'(x) > 0$. Assuming that the level of deductible $k$ is fixed by the insurance company and that the insurance company is unable to observe the level of self-protection exerted by homeowners, such that the premium is fixed at $\pi(x = 0)$. What will the expected utility maximizing level of self-protection be? Justify and explain your answer. If homeowners are again allowed to select the level of deductible $k$, will they choose the same amount as in the preceding question?

(d) Now allow for partial losses and consider some specific numeric values for the key parameters. Assume that the probability of a fire is equal to 0.10. A house is worth $100,000 and the deductible is fixed at $k = 0.50$, meaning that any losses up to $50,000 will be borne by the homeowner, and losses beyond this are paid by the insurer. If a fire occurs, the damage (a random variable) is uniformly distributed between $0$ (no loss) and $100,000$ (a complete loss). If a house has a fire, what is the expected level of damages from the fire? What is the probability that the insurance company will have to pay a claim? What is the actuarially-fair level of premium? (Show your work—write out the relevant equations and provide numerical solutions for each question.)

(e) Holding all parameters at the same levels as in the previous question, consider the problem in the case where the damage is distributed according to a symmetric triangular distribution. This distribution is given by the following figure (where values are given in $\$\text{thousands}$).
Provide numerical solutions for the same variables (expected loss, probability of loss, and actuarially fair premium) as in the preceding question.

(f) Finally, consider a more general case of an insurance company holding a large portfolio of policies having heterogeneous risks. The company cannot measure individual homeowners' risks, but knows that the average risk of a fire is 0.10. If the company sets the premium using this average probability of loss, what are the possible consequences?
3. In 1662, the English Parliament enacted the “Settlement Act,” which said that each person had a “parish of settlement,” meaning that each person “belonged” to a specific region (a parish) of the country. The Act gave local parish governments the right to remove any individual that did not belong in their region. The end result, according to Adam Smith, was a restriction on the mobility of labor across regions.

Consider the case of two neighboring parishes (A and B) for which all farm land in both parishes is owned by a single individual (the “lord”). Land quality is heterogeneous across the two different regions, such that it is more productive in region A than region B. Land is, of course, immobile among the parishes. The total amount of labor in the two regions is fixed. Food is the only output and is produced using only labor and land.

(a) Using a simple diagram, consider the effect of the Settlement Act (which prevented labor from moving among regions) on:
   - total returns to labor in both regions
   - total returns to the landowner
   - wages earned by workers in each region

(b) The Settlement Act was repealed in 1795 by the “Removal Act.” This allowed workers to move freely among regions. Consider the effects of this policy on the same set of variables.

(c) True or False—the landowner unambiguously benefitted from the repeal of the Settlement Act.

(d) How do the returns to the landowner depend on the productivity of labor in the two different regions?

(e) Now assume that a tax is put on the wages of workers in Parish A (but not in Parish B). What does this do to labor allocations and returns?

(f) Now assume that, instead of a single landowner owning all land, there are two landowners—one in Parish A and one in Parish B, each of which owns all the land in their parish. How did the repeal of the Settlement Act affect each landowner? Is it to the advantage of the two landowners to form an agreement to operate as a single entity?
4. Consider a firm that sells vacation packages that consist of a plane trip and a hotel stay. Consumers care only about the total cost of the vacation. The higher the price of a hotel room \( p_H \), the less people will be willing to pay for the airline ticket \( p_A \), and vice versa. Demand for vacations is given by:

\[
q(p_V) = 100 - p_V, \tag{1}
\]

where \( p_V \) is the price of a vacation, \( p_V = p_A + p_H \). The firm is the only firm selling this vacation (it is a monopoly). Each airline trip costs the firm \( c_A \) and each hotel stay costs \( c_H \).

(a) To begin, consider the case where the firm realizes that consumers care only about the price of a vacation, and so it chooses \( p_V \) in order to maximize profit. Provide an expression for profits and derive an expression for the profit-maximizing price.

(b) Now consider the case where the price of airline seats and hotel stays are set by separate divisions in the firm, each of which cares only about its own profits. In this case, the hotel division takes the price of the airline division as given and chooses \( p_H \) in order to maximize its profit. Likewise, the airline division takes the price of hotel stays as given and chooses \( p_A \) to maximize its profits. Solve for the reaction functions and optimal prices of each division as well as the optimal price of the entire tour package \( (p_V = p_A + p_H) \).

(c) In which case (the divisions operating independently or as a group) is the optimal price higher? Explain your answer.

(d) In which case are profits higher? Explain your answer.
5. A consumer only consumes one good, buying \( n \) units of it. The quality of the good \((q)\) matters to the consumer and each unit of quality costs \( p \). Each unit of \( n \), holding quality constant, costs \( pq \). The consumer has a fixed income of \( Y \), so that \( Y = pqn \). The consumer has the following utility function:

\[
U = \frac{n^\delta}{\delta} + \alpha \frac{q^\delta}{\delta}
\]

(2)

where \( \alpha \) is a constant.

(a) Solve for the first-order conditions for maximizing utility subject to the income constraint. At the point of tangency, what is the relative price for quantity \((n)\)? What is the relative price of quality \((q)\)?

(b) Solve for the elasticity of substitution between quantity and quality, holding utility constant.

(c) Solve for two "income" elasticities of demand for \( n \), holding the relative prices of \( q \) and \( n \) constant (that is, hold the slope of the utility curve constant). First, derive the elasticity of \( n \) with respect to \( U \) (this is the "utility" elasticity of \( n \)). Second, derive the elasticity of \( n \) with respect to \( Y \) (this is the money income elasticity of \( n \)).

(d) Draw the budget constraint and what \( U \) would have to look like for there to be an interior solution.

(e) An interior solution requires \( \delta < 0 \). Let \( \delta = -1 \). Solve for the Marshallian demand function for \( n \).

i. If income goes up 10%, how much will \( n \) go up in percent terms?

ii. Assume \( \alpha \) increases with income. Using the demand function, show that an increase in income can lead the consumer to reduce their demand for \( n \).

iii. Suppose that over time we observe that the quantity of cars bought goes down as people earn more. Does this mean that cars, adjusted for quality (that is, \( qn \)) are inferior goods?
6. A risk-neutral firm hires a risk-neutral worker for a job that requires a fixed level of effort. If the worker exerts that effort level, the worker produces value $V$ for the firm at a cost to the worker of $E$ (effort costs). If the worker supplies no effort—if the worker "shirks"—the worker produces a value to the firm of $0$ and has $0$ effort cost. The firm can monitor the worker at a cost of $C$. Only with monitoring can the firm tell if the worker is shirking. If caught shirking, the worker gets paid $0$; otherwise, the worker gets a wage of $W$. If the worker does not shirk, their full wage is $W - E$. Terms:

- $V =$ Value of worker produces if worker exerts effort
- $W =$ Wage (paid if not caught shirking)
- $C =$ Cost of monitoring
- $E =$ Effort Cost if worker exerts effort

(a) Write the pay-off matrix for the worker and for the firm, showing the results for the work/shirk choices and the monitor/not monitor choices.

(b) Assuming $V > W + C$ and $W > E$, and $E > C$, is there a dominant strategy?

(c) The worker chooses $p$, the probability they will work in a given period, and $1 - p$, the probability the worker shirks. Similarly, the firm chooses $q$, the probability the firm monitors the worker in a given period, and $1 - q$, the probability the firm does not monitor the worker. Describe the Nash equilibrium strategy for the worker and the firm. Derive the equilibrium values of $p$ and $q$. Assume the worker and the firm knows each other's values for $V, W, E$, and $C$.

(d) Let $P = W - E$ be the "premium" the firm pays the worker. Show the conditions under which the firm is better off paying a positive premium.
1. Consider a model economy where government expenditures, $\gamma$, affect the productivity of privately owned factors. Output per capita depends on the public expenditure on the public good as well as capital. That is,

$$y_t = A\gamma_t^\alpha k_t^{1-\alpha} \text{ where } 0<\alpha<1.$$ 

Public expenditure is financed by a proportional tax on income, $\tau$. The government cannot borrow; hence, it must always have a balanced budget.

As usual, the individuals maximize utility:

$$\int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{k}_t = (1-\tau)y_t - c_t - \delta k_t$$

a) Find the first order conditions of the individual’s maximization problem, and the dynamic equation for consumption in a competitive economy. What does the growth rate of consumption depend on (in equilibrium)?

b) Does this model exhibit endogenous growth in the long-run? Explain why it does or fails to do so.

c) How can the government maximize growth in this competitive economy? What happens when there are no taxes?

d) Solve the social planner’s problem to see if the competitive equilibrium is socially optimal. Discuss why the competitive equilibrium is or is not socially optimal.

2. Assume that the infinitely lived representative individual maximizes

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - \theta \log L_t \right] \right]$$

where $\beta$ is the discount factor, $L_t$ is the labor effort and $C_t$ is consumption. The individual supplies labor to the firms in return for the wage rate $w$, accumulates capital
and rents it out to the firms at rate \(1 + r^k\). Moreover, the individual holds government bonds \(B\), which has a net rate of return \(r^b\). While the government does not collect taxes from the return to bond holdings, they collect taxes from the return to capital. The tax rate is \(\tau\). Hence, the individual’s budget constraint is given by the following:

\[
C_t + K_{t+1} + B_{t+1} = w_t L_t + (1 + r^b_t)B_t + [1 + (1 - \tau)r^k_t] K_t.
\]

The production function of the perfectly competitive firm is:

\[
Y_t = (AL_t)^{\alpha}K_t^{1-\alpha}
\]

where \(A\) is the constant level of technology. Assume for simplicity that there is no depreciation, and the firm maximizes profits: \(\Pi_t = Y_t - (1 + r^b_t)K_t - w_t L_t\). Further assume that there is no growth in the steady state, therefore, technology, consumption, capital, bond holdings and output are constant in the steady state, and \(A = 1\) in the steady state. Government purchases consumption goods, \(G_t\), using the following rule: \(G_t = \frac{\zeta}{\kappa}z_t\), where \(\frac{\zeta}{\kappa} = g\) is the steady state government spending to capital ratio. \(z_t\) is a government spending shock that follows an AR(1) process: \(\dot{z}_t = \rho\dot{z}_{t-1} + \epsilon_t\), \(E_{t-1}(\epsilon_t) = 0\), and is equal to 1 in the steady state. Finally, the government runs a balanced budget in every period stabilizing:

\[
B_{t+1} = (1 + r^b_t)B_t + G_t - \tau r^k_t K_t
\]

a) Find the Euler equation for consumption and interpret it. Make sure to show all the first order conditions.

b) Obtain the first order conditions of the firm’s problem.

c) Find the steady state ratios for \(\frac{L}{K}\), \(\frac{Y}{K}\), \(\frac{B}{K}\) and \(\frac{G}{K}\) in terms of the parameters of the model.

d) What is the impact of \(\tau\) on the steady state allocation of the two types of capital?

e) The solution to this model can be obtained by solving the system of linear expectation equations in the endogenous variables and the exogenous variable, \(z\). Obtain the system of equations.

f) What is the necessary condition for the determinacy of this model? (Just write down the condition that applies to this model. You do not need to check if the condition holds.)

3. An agent lives in a Robinson-Friday type economy (trees produce fruits) with a very large number of other agents. The one-step-ahead equilibrium pricing kernel in this economy is \(q(x, x')\). At time \(t\), the agent is given a gift of a lottery ticket that has a nonnegative function \(w(x')\) written on it. The lottery ticket entitles its owner to receive \(w(x')\) units of consumption good at time \(t + 1\) contingent on \((x_{t+1} = x')\). The agent decides to sell claims to parts of her lottery ticket in order to increase her consumption
of time $t$ goods. She proceeds as follows. She plans to sell $B$ units of bonds bearing coupon rate $R$. Each bond will pay off at $t + 1$ as follows:

- payoff per bond if $w(x') \geq RB : R$
- payoff per bond if $w(x') < RB : w(x')/B$.

In addition to selling bonds, the agent sells equities in the amount of $S$ shares. Each unit of equities promises to pay off as follows

- payoff per share if $w(x') \geq RB : [w(x') - RB]/S$
- payoff per share if $w(x') < RB : 0$.

The agent chooses the coupon rate $R$ and numbers of bonds and shares, $B$ and $S$, to maximize the value of the lottery ticket. If we let $p_B(t)$ be the price of the bonds and $p_s(t)$ the price of the shares, the value of the lottery ticket in time $t$ goods is $p_B(t)B + p_s(t)S$.

1. Use an arbitrage argument to find formulas for $p_B(t)$ and $p_s(t)$ in terms of $q(x, x')$.
2. Find the value of the lottery ticket $p_B(t)B + p_s(t)S$ as a function of $B, S, R, w(x')$ and $q(x, x')$.
3. What values of $R, B$ and $S$ does the agent choose?

NOTE: The state in the economy is $x_t$; it evolves according to a Markov chain with $\text{prob}(x_{t+1} = x'|x_t = x) = \int_0^{x'} f(u, x) du$. Part (c) of this question is the celebrated Modigliani-Miller theorem in Finance.

4. The representative household chooses time paths for consumption, $c_t$, leisure, $l_t$, and real money balances, $m_t = M_t/P_t$, based on the welfare criterion:

$$W = \sum_{t=1}^{\infty} \beta^t u\left(c_t, l_t, \frac{M_t}{P_t}\right).$$

Here, $0 < \beta < 1$ and $P_t$ is the price level at time $t$. The only assets available to households are money, and physical capital. Given its current income, and any net transfers from the government ($g_t$), the household allocates its resources between consumption, investment, capital, and real money holdings. The government only prints money and makes transfers to households. Output is produced using capital and labor according to the technology

$$Y_t = e^{z_t} K_{t-1}^\alpha N_t^{1-\alpha}$$

with $0 < \alpha < 1$, $z_t = \rho z_{t-1} + \epsilon_t$ and $\epsilon_t$ has a standardized normal distribution; capital depreciates at the rate $\delta$. Note the timing convention in the technology function: the capital carried over from period $t - 1$, $K_{t-1}$, is available for use in producing goods during period $t$. A similar convention applies to money balances $M_t$. Let $\theta_t$ denote the growth rate of the nominal money supply. Assume then that the average growth rate is $\theta^{ss}$ and let $u_t = \theta_t - \theta^{ss}$ be the deviation in period $t$ of the growth rate from its unconditional average value. This deviation obeys the following stochastic process:

$$u_t = \gamma u_{t-1} + \phi z_{t-1} + \epsilon_t, \quad 0 \leq \gamma < 1,$$
where \( e \) is a white noise process. The total available time for work and leisure is 1. To simplify the problem you can assume that households know the value of \( z_t \) and \( u_t \) at the time they make their optimal decisions.

(a) What is the reason behind assuming that real money balances yield direct utility? (Answer in no more than 3 lines).

(b) Explain how the correlation between output, \( Y_t \), and inflation, \( p_{t+1}/p_t = 1 + \pi_{t+1} \), varies with \( \phi \). What value of \( \phi \) accords with your supply-demand intuition? (Answer in no more than 5 lines)

(c) What is the budget constraint faced by the representative household? Assume the household owns and operates the production function.

(d) Formulate the Bellman equation associated to the household’s problem. Hint: re-write the budget constraint in terms of the households’ financial wealth:

\[
\alpha_t = \tau_t + \frac{1}{1 + \pi_t} m_{t-1}
\]

where \( m_t = M_t/P_t \)

(e) Using the Bellman Equation derive the first order conditions for consumption, real money balances, labor, and capital. Note: you will not receive points if you use the Lagrangian.

(f) What are the derivatives of the Bellman Equation with respect to \( \alpha_t \) and \( k_{t-1} \)?

5. A worker receives, when unemployed, an offer to work forever at wage \( w \), where \( w \) is drawn from the distribution \( F(w) \). Wage offers are identically and independently distributed over time. The worker maximizes

\[
E \sum_{i=0}^{\infty} \beta^i u(c_i, l_i), \quad 0 < \beta < 1
\]

where \( c \) is consumption and \( l \) is leisure. Assume \( R_t \) is iid with distribution \( H(R) \). The budget constraint is given by

\[
a_{t+1} \leq R_t (a_t + w_t n_t - c_t)
\]

and \( l_t + n_t = 1 \) if the worker has a job that pays \( w_t \). If the worker is unemployed, the budget constraint is \( a_{t+1} \leq R_t (a_t + z - c_t) \). Here \( z \) is unemployment compensation. It is assumed that \( u(.) \) is bounded and that \( a_t \), the worker’s asset position, cannot be negative.

(a) Write down the Bellman’s equation for this problem

(b) What is the optimal strategy for the worker? An argument outlining the strategy and how this depends on the model’s structure is enough (No more than five lines).
Instructions. Read carefully and answer all questions. If you find an error/typo in the text, state it and solve the fixed question.

1. Consider the following economy. There is only one consumption good, which cannot be produced or stored. The total amount of goods available at each period is constant at $N$. There are $2N$ households, divided into equal number $N$ of two types, according to their endowment sequences. The two types of households, dubbed odd or even, have endowment sequences:

$$\{y_{t}^{o}\}_{t=0}^{\infty} = \{1,0,1,0,1,0,...\},$$

$$\{y_{t}^{e}\}_{t=0}^{\infty} = \{0,1,0,1,0,1,...\}.$$  

Households of both types order consumption sequences $\{c_{t}\}$ according to the common utility function

$$U = \sum_{t=0}^{\infty} \beta^{t} u(c_{t}),$$

where $\beta \in (0,1)$ and $u(\cdot)$ satisfies the usual Inada conditions.

(a) Pareto Allocation: Let $\theta \in [0,1]$ denote a weight indexing how much a social planner likes odd agents. Find the optimal consumption allocations as a function of the parameter $\theta$ assuming that the central planner maximizes the weighted utility function. What are the allocations if $u(c) = c^{1-\beta}/(1-\beta)$?

(b) Complete Markets: Suppose households have access to Arrow-Debreu securities whose price is $q^{0}$. Furthermore, households take a price sequence $\{q^{0}_{t}\}$ as given and chooses a consumption sequence to maximize $\sum_{t=0}^{\infty} \beta^{t} u(c_{t})$ subject to the budget constraint. State the budget constraint and derive the optimality condition for consumption. Distinguish between the cases $c_{t} = 0$ and $c_{t} > 0$. Define a competitive equilibrium for this economy. Find $q^{0}_{t}$ and the consumption bundles for each type of household. What is the weight on odd households that equate the complete markets allocation with that of the Central Planner?

(c) Ricardian Proposition: Now suppose we add a government to the economy described in section (6). This government levies lump-sum taxes on agents of type $i = o,e$ at time $t$ of $r_{i}^{t}$. The proceeds are used to purchase a constant level of government expenditure $G \in (0,1)$ each period $t$. Re-write the households' budget constraint.

State the government's budget constraint. Define a competitive equilibrium for the modified economy. Compute the new equilibrium allocations and price sequence as a function of $r_{i}^{t} = \sum_{t=0}^{\infty} q^{0}_{t} r_{i}^{t}$. Show that the equilibrium displays Ricardian Equivalence.

(d) Fiat Money: Endowments and utility functions are the same as before but now household don't have access to complete markets. Instead, at time $0$, the government endows each of the $N$ even agents with $M/N$ units of unbacked or inconvertible currency. Odd agents are initially endowed with zero units of the currency. Let $p_{t}$ denote the time $t$ price level, denominated in dollars per time $t$ consumption good. We seek an equilibrium in which currency is valued ($p_{t} < \infty$ for all $t$) in which each period agents not endowed with goods pass currency to agents who are endowed with goods. Contemporaneous exchanges of currency for goods are the only exchanges that are permitted.

State the households' optimization problem including their budget constraints. What are the first-order conditions? Find the price level $p_{t}$ as a function of the structural parameters and the initial consumption level by odd households $c_{0}$. HINT: Look for a constant price level.

(e) Taxes and Money: Consider the same environment as in point (d). Let the government's budget constraint be $M_{t} = (1+\kappa) M_{t-1}$. At time $t$, the government transfers or taxes nominal balances in amount $\kappa M_{t-1}/(2N)$ to each household of each type. The total transfer at time $t$ is thus $\kappa M_{t-1}$, because there $2N$ households receiving transfers.
Show that the price level at time 0 obeys
\[(1 - c_0) p_0 = \frac{M_{-1}}{N} \left( 1 + \frac{T}{2} \right),\]
where \(c_0\) is the initial consumption level by odd households and \(M_{-1}\) is the stock of money at time \(-1\). HINT: Guess and verify that \(M_t/p_t = k\), where \(k\) is some constant.

2. An unemployed worker receives every period an offer to work forever at the wage \(w\), where \(w\) is drawn from the distribution \(F(w)\). Offers are independently and identically distributed. Every agent has another source of income, which we denote \(\varepsilon_t\), and that may be regarded as nonhuman wealth. In every period all agents get a realization \(\varepsilon_t\), which is independently and identically distributed over time, with distribution \(G(\varepsilon)\). We assume that \(w\) and \(\varepsilon\) are orthogonal. A worker wants to maximize
\[E \sum_{t=0}^{\infty} \beta^t y_t \quad 0 < \beta < 1,\]
where \(y_t = w + \phi \varepsilon_t\) if the worker has accepted an offer that pays \(w\), and \(y_t = c + \varepsilon_t\) if the worker remains unemployed. We assume that \(0 < \phi < 1\) to reflect the fact that an employed worker has less time to engage in the collection of nonhuman wealth. Assume \(0 < \text{prob}(w \geq c + (1 - \phi)\varepsilon) > 0\).

(a) Analyze the worker's problem. Write down Bellman's equation.

(b) Show that the reservation wage increases with the level of nonhuman wealth, i.e., wealthier workers are less willing to accept low-paying jobs. Provide intuition as to why \(\phi\) must be below 1 for the result to hold. HINT: Instead of using derivatives, evaluate the worker problem assuming her nonhuman wealth has increased by \(\Delta\).

For the rest of the problem assume workers have no nonhuman wealth, i.e., \(\varepsilon = 0\) for all \(t\). Instead, assume that workers receive each period an offer to work for wage \(w_t\) forever, where \(w_t = w\) in the first period and \(w_t = \theta \varepsilon\) after \(t\) periods in the job. Assume \(\theta > 1\), that is, wages increase with tenure. The initial wage offer is drawn from a distribution \(F(w)\) that is constant over time; successive drawings across periods are iid. Let \(v(w)\) be the optimal value of the objective function for an unemployed worker who has offer \(w\) in hand.

(c) Write Bellman's equation for this problem.

(d) Argue that if two economies differ only in the growth rate of wages of employed workers, say \(\theta_1 > \theta_2\), the economy with the higher growth rate has the smaller reservation wage. For this part assume \(\theta_i \theta < 1\) for \(i = 1, 2\). HINT:
\[\frac{\partial}{\partial \omega} \int_{\omega}^{\infty} (w' - \omega) F(\omega') \, d\omega' < 0.\]
Consider a model economy where output of a firm depends on aggregate amounts of capital and labor in addition to the factors of production employed by each firm. That is,

\[ Y_{it} = A(K_{it}^\alpha L_{it}^{-\beta}) K_{it}^\alpha L_{it}^{-\alpha} \text{ where } \alpha > 0, \beta > 0. \]

\( Y_{it} \) is output produced by the representative firm, \( K_{it} \) and \( L_{it} \) are the amount of capital and labor the firm uses in production, and \( K_t \) and \( L_t \) are their aggregate counterparts. The competitive firm maximizes profits: \( \Pi_{it} = Y_{it} - (r_t + \delta) K_{it} - w_t L_{it} \), where \( r \) is the rental rate of capital, \( \delta \) is the depreciation rate and \( w \) is the wage rate.

As usual, the individuals maximize utility:

\[ \int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \]

subject to

\[ \dot{k}_t = w_t + r k_t + \eta_0 - c_t \]

where \( y_t, c_t, k_t \) are output, consumption and capital per capita, respectively, and \( \eta_0 \) is the initial wealth. Assume for simplicity that there is no population growth.

a) Solve for \( r \) and \( w \) in terms of \( k \) using the firm's first order conditions and the fact that each firm will choose the same capital per labor ratio in equilibrium.

b) Set up the Hamiltonian and obtain the first order conditions of the individual's maximization problem, and the dynamic equation for consumption in the competitive economy.

c) Under what special condition will this model exhibit endogenous growth? Explain the rationale behind that condition. What does the growth rate depend on?

d) Solve for the consumption along the optimal balanced growth path.

4. Assume that the infinitely lived representative individual maximizes

\[ E_0 \left[ \sum_{t=0}^\infty \beta^t [\log(C_t) - \theta \log(L_t)] \right] \]
where $\beta$ is the discount factor, and $\theta$ is a positive constant. The individual supplies labor to the firms in return for the wage rate $w_t$, accumulates capital and rents it out to the firms, and holds government bonds that. Hence, the individual's budget constraint is given by the following:

$$C_t + K_{t+1} + B_{t+1} = X_t K_t + w_t L_t + R_t B_t - T_t,$$

where $X$ and $R$ are the gross returns on capital and bond, respectively, and $T$ is the lump-sum tax the individual pays to the government. The production function of the perfectly competitive firm is:

$$Y_t = (AL_t)^{\alpha} K_t^{1-\alpha}$$

where $A$ is the constant level of productivity. Assume for simplicity that there is no depreciation, and the firm maximizes profits: $\Pi_t = Y_t - X_t K_t - w_t L_t$.

There is no growth in the steady state, therefore, technology, consumption, capital and output are constant in the steady state. Assume that $A = 1$ in the steady state. The government finances its spending by collecting lump-sum taxes and issuing bonds. Therefore, the government balanced budget rule is given by:

$$B_{t+1} + T_t = G_t + R_t B_t,$$

where $G_t$ is the government spending, which is subject to shocks.

a) Find the Euler equation for consumption and interpret it.

b) Find the firm's first order conditions.

c) Assume that the steady state tax to labor ratio is given and is $\tau$. Find the steady state ratios for $K_t / L_t$, $Y_t / L_t$, $C_t / L_t$, $B_t / L_t$ and the wage rate and the gross returns to assets in terms of the parameters of the model.

d) Obtain the system of linearized equations. In obtaining the system, use the following assumptions:
   - the percentage deviations of $G_t$ from its steady state value follow an AR(1) process: $\hat{G_t} = \phi \hat{G}_{t-1} + \epsilon_t$, $E_t(\epsilon_t) = 0$
   - consumption and the rental rate of capital are jointly log-normal and homoskedastic. (Hint: if $X$ and $Y$ are jointly log normal, then $\log \{ E_t(X_{t+1}Y_{t+1}) \} = E_t [\log (X_{t+1}Y_{t+1})] + \frac{1}{2} \text{var}_t [\log (X_{t+1}Y_{t+1})]$.

e) What is the necessary condition for the determinacy of this model?

f) Does this model exhibit precautionary savings? Explain.
Micro Preliminary Exam Winter 2009

1. **True, False, Uncertain:** For each question, state whether it is true, false, or uncertain. Explain your answer. Grading of these questions depends mainly on the quality of your answers.

   a. A consumer consumes 3 goods: good 1, good 2, and good 3. The share of each good in spending is 1/3. The Hicksian cross-elasticity of substitution between good 1 and good 2 is zero. The Hicksian own-price elasticity of demand for good 1 is -1 and the Hicksian own-price elasticity of demand for good 2 is -2. The own-price elasticity of demand for good 3 is -3.

   b. A city buys a tract of homes in order to tear them down and build a parking lot. The city required homeowners to sell their homes to the city. If the city pays the prevailing market price for homes (including compensation for the time and cost of moving to a new home), then the homeowners are no worse off.

   c. A price ceiling on a good with perfect inelastic supply will have no effect on economic welfare.

   d. The government mandates that all farmers use less fertilizer. This will reduce the cost of farming.

   e. A consumer consumes two goods, X and Y, spending half of his or her income on each good. Good X and Y are perfect complements. Both the Hicksian Price Elasticity of Demand for Good X and the Marshallian Price Elasticity for Good X are equal to one.

   f. A fall in the cost of homes will increase the price of furniture. (Assume the normal competitive supply and demand curves).
2. In order to regulate growth, the City of Raleigh imposes an "impact fee." This is a fee of $X that must be paid on every new home that is built. The fee does not apply to non-residential structures (e.g., stores and office buildings) and does not apply to existing homes. Consider the effects of this plan and answer the following questions. Illustrate and reinforce your answers with the appropriate economic theory. In answering the question, you can ignore any change in services provided by the city (i.e., assume the tax revenues are unspent) and assume that construction is a competitive industry. You can also assume that the city is "closed," meaning that people's decisions of whether to enter or leave the city are not affected by the impact fee.

(a) Describe the effects of this program on the following and consider how the elasticities of supply and demand influence your answers:

i. New home prices
ii. Existing home prices
iii. The price of non-residential structures
iv. The number of new homes built
v. Residential rents (i.e., the implicit dollar value of the annual services from a home)
vI. The welfare of new home buyers
vii. The welfare of existing home owners
viii. The welfare of home builders
ix. The wages of construction workers

(b) How would your answer change if the plan is announced today but will not be implemented until 2010?

(c) How would your answer change if the plan is implemented now but it is also announced that it will end in 2015?

(d) How would your answer change if there is considerable uncertainty about whether the plan (to permanently tax all new homes by $T) will stay in place in the future if home owners are risk-adverse?

(e) Finally, consider the effects of the program if instead of applying the fee to new homes, it is applied to residential rents (on all homes—new or used) in the amount of $Y per month.
3. **Optimal Taxation**

Assume that the Hicksian demand curves are linear in the relevant range. For good \( i \), the demand curve is:

\[
p_i = a_i - b_i x_i,
\]

where \( x_i \) is the quantity of good \( i \) consumed, \( p_i \) is the price paid by consumers, and \( a_i \) and \( b_i \) are constants (within the relevant range of this problem). Next, assume the supply of good \( i \) is infinitely elastic at price \( = c_i \).

The economy is competitive, so that if \( t_i \) is the per unit tax on good \( i \), then in equilibrium:

\[
p_i = c_i + t_i.
\]

a. Draw the demand and supply curve for good \( i \). Shade in the deadweight loss of the unit tax \( t_i \). Let \( x_i^* \) be the quantity of \( x_i \) if there is no tax and \( x_i' \) be the quantity after the tax.

b. Write the absolute change in \( x \) \( (x_i' - x_i^*) \) as (1) a function of the slope \( (b_i) \) and the tax \( t_i \). Next write the corresponding expression for the deadweight loss. Finally, write the deadweight loss as a function of the tax rate \( \tau_i \) \( (\tau_i = t_i/c_i) \).

c. Suppose there are three goods and only good 1 and good 2 are taxed. Good 3 is not taxed. Further, assume that the Hicksian cross-price elasticities between good 1 and good 2 is zero. Let the total taxes collected on good \( i \) be \( T_i \) \( (= \tau_i c_i x_i) \).

Write the LaGrangian for the following problem. The government wants to collect \( \$G \) in total taxes and it can only use the above unit taxes (on goods 1 and 2) to collect tax revenues. It wants to minimize the total deadweight loss of the taxes it imposes subject to the constraint that it wants to collect \( \$G \) in taxes. (All persons in this economy have exactly the same demand curves and incomes). Use the tax rates \( (\tau_1 \text{ and } \tau_2) \) as the variables.

d. Solve for the optimal ratio of \( \tau_1/\tau_2 \). Next, using the Hicksian demand curve, solve for the price elasticity of demand for good \( i \) (in absolute value, \( \eta_i \)) at price \( c_i \). Express the ratio of tax rates as a function of the price elasticities of demand.

f. If good 1's Hicksian price elasticity is twice that of good 2, what should the ratio of tax rates be?
4. **Question: Properties of Utility Function**

A consumer has the following utility function:

\[ U = (x - x_0)^\alpha (y - y_0)^{1-\alpha} \]

where \( x_0 \) and \( y_0 \) are constants and can be thought of as the minimum amount of goods that the consumer needs to survive and \( 0 < \alpha < 1 \). The consumer spends all their income \( (M) \) and goods \( x \) and \( y \); \( p_x \) is the price of \( X \) and \( p_y \) is the price of \( y \). For convenience, in the equations below, you can use the following notation:

- Subsistence Income = \( S = p_x x_0 + p_y y_0 \).
- \( \bar{x} = x - x_0 \), \( \bar{y} = y - y_0 \).

a. Explicitly derive the Hicksian (compensate) demands for good \( x \). (By explicitly, we mean that you should show all work. For the questions below, you can refer back to previously derived results).

b. Explicitly derive the Marshallian (uncompensated) demands for good \( x \).

c. Demonstrate that A and B satisfy the own-price versions of the Slutsky equation for good \( x \).

d. Explicitly derive the indirect utility function corresponding to this utility function. When will doubling all goods double utility?

e. Suppose the above equations describe the relationship between \( x \) and income (holding prices constant). Discuss what the coefficients of the following regression should look like:

\[ x^D = a + b M. \]

Suppose someone ran the following regression:

\[ x^D = a + dM^2. \]

What should the error pattern look like?
During World War II in the U.S., many consumer goods were rationed. Each person had a fixed weekly “income” of ration points, apart from any money income, and each rationed commodity had a “price” fixed in terms of ration coupon points apart from its price in terms of money. To buy a unit of a commodity, a consumer would have to pay both its money price and its ration coupon point price. Consider the case of two commodities—“food” and an aggregate commodity representing “all other goods.” Both commodities are rationed and thus have both a monetary price and a coupon point price. Ration coupons are generic and can be used to purchase either food (at its coupon price) or the all other goods composite (at its coupon price). It is illegal to buy or sell ration coupons.

a. Set up the consumers’ optimization problem and draw a diagram that illustrates the optimal solution to the problem. Interpret the solution to the problem when only one of the constraints is binding. Interpret the solution when both constraints are binding.

b. In December 1943, the ration point price of food was reduced. This “ration price reduction” was intended to cause an increase in purchases of food. Yet it caused also an increased consumption of the “all other goods” composite commodity, although the ration point value of other goods was left unchanged. Illustrate this change. Does this suggest that food and “all other goods” are complements? Why or why not?

c. Now consider a slightly different rationing system. Assume that food is the only good that requires coupons. In the absence of any policy, the equilibrium price of food is $P/unit. The government wishes to ensure a minimum level of nutrition, and so purchases of food up to 100 units are subsidized such that consumers only pay $P/2 per unit. All subsequent units must be purchased at the price of 2*$P per unit. Show the initial budget constraint and the post-policy budget constraint. On separate diagrams, show a consumer that buys more and one who buys less food after the policy.

d. If the policy in part (c) leaves a consumer no better or no worse than the case of free trade in food (i.e., no policy at all), is the consumer going to consume more or less food than in the case of no policy? Demonstrate your answer with a diagram.
6. The US is a large exporter of wheat. Being a "large" exporter means that it faces a downward sloping demand function for its wheat exports to the rest of the world. The wheat industry is perfectly competitive and has free entry and exit. The US government is considering a number of policy measures intended to benefit its wheat growers. Consider each intervention measure listed below and identify the economic\welfare impacts on US producers of wheat, US consumers of wheat, and the US Treasury (i.e., taxpayers). You should identify any effects on domestic US and world prices, on the amounts consumed and produced, and on the level of exports. Your answers should include illustrations as appropriate.

a. An open-ended production subsidy that establishes a fixed support price (call it \P_{\text{Support}}\) and pays any difference between the support price and the prevailing market price with a payment from the US Treasury.

b. A subsidy of \$S that is paid on each unit of wheat that is exported.

c. A production quota that cuts the amount of wheat produced to one-half of the amount being produced prior to the imposition of the policy.

d. A subsidy of \$s that is paid on each unit of wheat that is consumed by US consumers.

Now answer the following questions.

e. In considering a policy measure that benefits wheat producers, discuss how the benefits to a wheat grower might differ if the grower owns no land but rather rents all of her land from a non-farming landowner.

f. Now think about a new farmer, who buys a farm after the policy is in place in order to enjoy the benefits of the policy. Will this second generation farmer enjoy the same level of benefits as the first generation of farmers? Why or why not?
1. Consider a model economy where output of a firm depends on two types of capital and labor in addition to the factors of production employed by each firm. That is,

\[ Y_t = AK_t^\alpha Z_t^\eta L_t^{1-\alpha-\eta} \]  

where 1 > \alpha > 0, 1 > \eta > 0.

\( Y \) is output produced by the representative firm, \( K \) and \( Z \) are the two types of capital, and \( L \) is the labor the firm uses in production. The competitive firm rents both types of capital and labor services from the households, and maximizes profits. The representative household consumes the homogeneous good \( Y \), provides labor services to the firms in return for a wage rate, \( w \), and accumulates assets. The problem of the household is to maximize life-time utility

\[ \int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \]

subject to

\[ \dot{a}_t = w_t + r_t a_t + a_0 - c_t \]

where \( c_t, a_t \) and \( r_t \) are consumption per capita, assets per capita and net return to assets, respectively. \( a_0 \) is the initial wealth. The assets are distributed between the two types of capital; hence \( a_t = k_t + z_t \). Assume for simplicity that there is no population growth and that the two types of capital depreciate at the same rate, \( \delta \).

a) Set up the Hamiltonian and obtain the first order conditions of the individual’s maximization problem, the transversality condition and the dynamic equation for consumption in the competitive economy.

b) Set up the firm’s problem and find the demand for labor and the two types of capital.

c) Under what special condition will this model exhibit endogenous growth? Explain the rationale behind that condition. What does the growth rate depend on?

d) Solve for consumption along the optimal balanced growth path under endogenous growth.
1. Consider a model economy where output of a firm depends on two types of capital and labor in addition to the factors of production employed by each firm. That is,

\[ Y_t = AK_t^\alpha Z_t^\eta L_t^{1-\alpha-\eta} \text{ where } 1>\alpha>0, 1>\eta>0. \]

\( Y \) is output produced by the representative firm, \( K \) and \( Z \) are the two types of capital, and \( L \) is the labor the firm uses in production. The competitive firm rents both types of capital and labor services from the households, and maximizes profits. The representative household consumes the homogeneous good \( Y \), provides labor services to the firms in return for a wage rate, \( w \), and accumulates assets. The problem of the household is to maximize life-time utility

\[
\int_0^\infty \frac{c_{t+1}^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt
\]

subject to

\[ \dot{a}_t = w_t + \tau_t \dot{a}_t + a_0 - c_t \]

where \( c_t, a_t \) and \( \tau_t \) are consumption per capita, assets per capita and net return to assets, respectively. \( \eta_0 \) is the initial wealth. The assets are distributed between the two types of capital; hence \( a_t = k_t + z_t \). Assume for simplicity that there is no population growth and that the two types of capital depreciate at the same rate, \( \delta \).

a) Set up the Hamiltonian and obtain the first order conditions of the individual’s maximization problem, the transversality condition and the dynamic equation for consumption in the competitive economy.

b) Set up the firm’s problem and find the demand for labor and the two types of capital.

c) Under what special condition will this model exhibit endogenous growth? Explain the rationale behind that condition. What does the growth rate depend on?

d) Solve for consumption along the optimal balanced growth path under endogenous growth.
2. Assume that the infinitely lived representative individual maximizes

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - \frac{L_t^\gamma}{1-\gamma} \right] \right] \]

where \( \beta \) is the discount factor, \( \chi \) is a positive constant and \( \gamma \) is the inverse of the elasticity of labor. The individual supplies labor to the firms in return for the wage rate \( w_t \), and accumulates capital and rents it out to the firms. In addition to consumption and investment decisions, the individual needs to choose the rate of capacity utilization; i.e., the fraction of capital they will be renting out in that period, \( u_t \). Hence, the individual's budget constraint is given by the following:

\[ C_t + I_t = R_t (u_t K_t) + w_t L_t \]

where \( R \) is the rental rate of capital, and \( w_t \) is the wage rate. The rate of utilization increases capital's depreciation, and the depreciation as a function of the rate of utilization is \( \frac{dK}{d\phi} \), where \( \phi \geq 1 \). As usual, investment augments the capital stock:

\[ K_{t+1} = I_t + \left( 1 - \tau \frac{dK}{d\phi} \right) K_t. \]

The perfectly competitive firm takes the rental rate of capital, the wage rate and the rate of utilization as given, and chooses capital and labor in order to maximize the profits. The firm's production function is given by:

\[ Y_t = A_t (u_t K_t)^{\alpha} L_t^{1-\alpha} \]

where \( A \) is the productivity shock. There is no growth in the steady state, therefore, technology, consumption, capital and output are constant in the steady state. Assume that \( A = 1 \) in the steady state.

a) Find the Euler equation for consumption and interpret it.
b) Find the firm's first order conditions.
c) Find the steady state values of \( \frac{Y}{K} \), the wage rate, the rental rate of capital and the utilization rate in terms of the parameters of the model.
d) Obtain the system of linearized equations assuming that the percentage deviations of \( A_t \) from its steady state value follow an AR(1) process.
e) What is the necessary condition for the determinacy of this model?
f) Show that under certain parameter restrictions, consumption follows a random walk (state the parameter restrictions).
Instructions. Read carefully and answer all questions. If you find an error/typo in the text, state it and solve the fixed question.

3. An unemployed worker receives each period a zero offer (or no offer) with probability \(1 - \pi(e)\). With probability \(\pi(e)\) the worker draws an offer \(w\) from the distribution \(F\). Here \(e\) stands for effort - a measure of search intensity - and \(\pi(e)\) is increasing in \(e\). A worker who accepts a job offer can be fired with probability \(\alpha, 0 < \alpha < 1\). The worker chooses a strategy, that is, whether to accept an offer or not and how much effort to put into search when unemployed to maximize

\[
E_0 \sum_{t=0}^{\infty} \beta^t y_t, \quad 0 < \beta < 1,
\]

where \(y_t = w\) if the worker is employed with wage \(w\) and \(y_t = 1 - e + z\) if the worker spends \(e\) units of leisure searching and does not accept a job. Here \(z\) is unemployment compensation. For the worker who searched and accepted a job, \(y_t = w - e - T(w)\); that is, in the first period the wage is net of search costs. Throughout, \(T(w)\) is the amount paid in taxes when the worker is employed. We assume that \(w - T(w)\) is increasing in \(w\). Assume that \(w - T(w) = 0\) if \(w = 0\), that, if \(e = 0, \pi(e) = 0,\) and that \(\pi'(e) > 0, \pi''(e) < 0\).

(a) What is the economic interpretation of \(\pi(e)\) being an upward sloping function? What is the intuition behind the assumptions \(\pi(0) = 0\) and \(\pi''(e) < 0\)? (Note: a prime denotes a derivative)

(b) Analyze the worker’s problem. Establish that the optimal strategy is to choose a reservation wage. Display the condition that describes the optimal choice of \(e\), and show that the reservation wage is independent of \(e\). HINT: Note that the state variable for this problem is \((w, e, s)\) where \(s = E\) if the worker is employed and \(s = U\) if unemployed. Let \(Q\) denote the expected value of the objective function for an unemployed worker who behaves optimally before getting an offer. Let \(v(w, s, E)\) be the objective function of an employed worker; \(v(w, e, U)\) is the value of an unemployed worker who has an offer \(w\) in hand and spends \(e > 0\) units of leisure searching this period. To answer the question, use \(v(w, e, U)\) to formulate the worker’s problem and find the reservation wage as a function of \(Q\) and the structural parameters in the model. Next, let \(\Phi(e) = \int_0^\infty v(w, e, U) F(dw)\), express this integral as a function of \(Q\) and the other parameters. Finally, show that \(\Phi(0) = 1 + z + \beta Q\) and use the values \(\Phi(e)\) and \(\Phi(0)\) to formulate the maximization problem that an unemployed worker uses to decide how much effort to put in the search.

4. Consider the following optimal taxation problem. There is no uncertainty. There is one good that is produced by labor of the representative household, and that can be divided among private consumption \(c_t\) and government consumption \(g_t\) subject to \(c_t + g_t = 1 - x_t\). The good is produced by zero-profit competitive firms that pay the worker a pretax wage of 1 per unit of \(1 - x_t\). A representative consumer maximizes

\[
\sum_{t=0}^{\infty} \beta^t u(c_t, x_t)
\]
subject to the budget constraint \( c_t + q_t b_{t+1} \leq (1 - \tau_t) (1 - x_t) + b_t \), where \( q_t \) is the price of consumption at \( t + 1 \) in units of time \( t \) consumption, and \( b_t \) is a stock of one-period bonds owned by the household. Here \( \tau_t \) is a flat-rate tax on the household’s labor supply \( 1 - x_t \). Assume \( u(c_t, x_t) = c - 0.5 (1 - x_t)^2 \).

(a) Does the production function display decreasing returns to scale? Why?

(b) Carefully display a competitive equilibrium for this economy. Show that \( q_t = \beta \) and \( x_t = \tau_t \).

(c) What is the transversality condition for the competitive equilibrium to be well defined? What is its interpretation?

(d) Assuming \( b_0 = 0 \) and using the transversality condition from the previous literal find the single intertemporal constraint for this economy.

(e) Assume \( g_t = 0 \) if time is even and \( g_t = 0.2 \) if time is odd and \( \beta = 0.95 \). What is the optimal tax rate on labor?

(f) Now assume \( g_t = 0 \) if time is odd and \( g_t = 0.2 \) if time is even and \( \beta = 0.95 \). What is the optimal tax rate on labor?

(g) What are the implications of (e) and (f) for taxation on labor?

(h) Under what conditions, if any, would \( \tau_t = 0 \)?
1. Policy Question

Many states tightly regulate sales of alcoholic beverages. One form of regulation is a liquor license. In order to sell liquor, an establishment must have a liquor license. The number of licenses is tightly controlled. An auction is held annually and the licenses are sold by the state. Licenses can also be sold or rented in a secondary market. Few bar owners actually own the licenses; rather they are typically purchased by investors and then rented or sold to bar owners. Using simple diagrams, answer the following questions about the rental and sales market for liquor licenses. Be sure to identify the welfare effects of the license policy and any changes to it in each case. In answering, ignore any externalities associated with alcohol consumption.

(a) Illustrate how the equilibrium rental price (assume an annual rental contract) is determined.

(b) Continuing from the previous question, illustrate how the equilibrium sales price of a license in the auction is determined.

(c) If the license policy were to be eliminated, what would happen to the price of an alcoholic drink? How would this affect the welfare of consumers, bar owners, and license owner/investors.

(d) What happens to the rental price of licenses if the state regulatory authority imposes a price ceiling that sets a maximum allowable price on alcoholic beverages?

(e) Liquor licenses cannot be traded across county lines. Currently, liquor licenses are issued in proportion to the county's population: for example, a big county that has twice the population of a small county will have twice as many licenses. Suppose that small counties have relatively more drinkers per capita than large counties. In which counties will liquor licenses be worth more? If costs are similar across counties, what will happen to the variance in the price of liquor licenses if licenses can be traded across county lines?

(f) True, False, or Uncertain: The liquor license policy benefits investors and not bar owners.
2. Production Theory Question

The transcendental logarithmic or “translog” cost function has become the work horse of applied, empirical production analysis. The cost function is commonly represented as:

\[
\ln C(w, y) = \alpha_0 + \alpha_y \ln y + \sum_i \alpha_i \ln w_i + \frac{1}{2} \gamma_{yy} (\ln y)^2 + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln w_i \ln w_j + \sum_i \gamma_{iy} \ln w_i \ln y
\]

where \( w_i \) is the price of input \( i \), \( y \) is total output, and the \( \alpha \) and \( \gamma \) terms represent parameters of the cost function. For this cost function, answer the following questions:

(a) For the two input case \( (i = 1, 2) \), solve for the factor demand equations.

(b) Solve for factor demand elasticities and the Allen-Uzawa elasticities of substitution for the two good case. (Note: The Allen-Uzawa elasticities of substitution are given by \( \sigma_{ij} = (CC_{ij})(C_i C_j) \) where \( C = \) is total cost and \( C_i \) is \( \partial C / \partial w_i \) and so forth).

(c) To be consistent with theory, cost functions must satisfy a number of conditions. Outline the conditions (in terms of restrictions on the parameters) that are required to satisfy the following conditions:

- Homogeneity (of degree one)
- Symmetry of the elasticities of substitution
- Constant returns to scale

(d) Inputs are characterized as substitutes or complements on the basis of the sign of the factor demand elasticities. Can an input be a complement for all other inputs in the translog cost function?

(e) True, False, or Uncertain: Symmetry of the elasticities of substitution implies symmetry of cross-price factor demand elasticities.

(f) The Cobb-Douglas cost function is a special case of the translog that is obtained when \( \gamma_{ij} = 0 \) for all \( i \neq j \). What does this imply about the elasticities of factor demands and substitution?
3. **General Equilibrium Question**

1. In a general exchange economy, there are only two types of consumers (A and B) and two goods (x and y). The two types of consumers have the same utility function:
   \[ u(x_i, y_i) = x_i \cdot y_i, \quad i = A, B \]

   They have different initial endowments of goods.
   - A starts with 8 units of x and 2 units of y
   - B starts with 2 units of x and 8 units of y

   (a) Draw an Edgeworth box for this economy when there in only one of each type of consumer. Characterize as precisely as possible the set of allocations in the core of this economy.

   (b) Continuing with (a), what are the Pareto efficient allocations?

   (c) Assume there are sufficient number (and equal number) of each type so that each consumer takes the price as given. Describe the Walrasian equilibrium.

   (d) What is the core of the economy in case (c)?

2. In the above economy, assume the two types of consumers have the following utility function:
   - For A: \[ u = x + y \]
   - For B: \[ u = x^2 + y^2 \]

   They have the same initial endowments of goods as in 1.
   - A starts with 8 units of x and 2 units of y
   - B starts with 2 units of x and 8 units of y

   (a) Draw an Edgeworth box for this economy when there in only one of each type of consumer. Characterize as precisely as possible the set of allocations in the core of this economy.

   (b) Continuing with (a), what are the Pareto efficient allocations?
4. Contract Theory

A landlord owns a building which Mr. Adams wishes to rent for a factory. If Mr. Adams rents the building, then he will spend this year preparing for production, at a cost of \( N \) dollars. If he remains in the building next year, he will sell his production and his profit will be \( P - N \) (assume all dollars have already been translated into present values). However, if the landlord kicks Mr. Adams out at the end of the first year, he will have paid \( N \) dollars but will receiving nothing. This is the only building suitable for Mr. Adam's purposes.

Normally, the landlord rents the building with a one-year lease. This year, he would rent to another possible tenant (Mr. Brown) for \( R \) dollars. The landlord does not know who will rent next year. With a probability of \( 0.5 \) Mr. Adams will be the only tenant; with a probability of \( 0.5 \) another tenant, Mr. Charles, will appear who will be willing to rent for \( Z \) dollars. Suppose all persons are risk-neutral.

Review:

Mr. Adams

| First Year | Cost of Preparing Building $N |
| Second       | Produce and Make Profit of $P - N |

Alternative

| First Year | Rent to Mr. Brown for $R |
| Second Year | 0.5 that Mr. Charles will appear and pay $Z |
|            | (Otherwise, Mr. Adams will rent the building) |

1) Suppose parties are able to negotiate an efficient outcome. Derive the necessary and sufficient conditions on the numbers \( P, N, R, \) and \( Z \) so that Mr. Adams will rent the building and prepare it for next year, but will be kicked out next year if a better tenant comes along.

2) Suppose the landlord can only offer one-year leases and cannot commit to next year's least until next year. He does not negotiate: he simply makes a take-it-or-leave-it offer to prospective tenants. In this case, who will rent this year? Next Year? Will Mr. Adams make preparation for production?

3) Suppose the landlord can commit to a 2 year lease, but that once the lease is written, it cannot be broken. Assuming it is impossible for tenants to lease to other tenants, given the conditions on \( P, N, R, \) and \( Z \) such that Mr. Adams will sign the two year contract? Will the outcome differ if tenants can rent to other tenants?
Consider the two-player game Work-LaZe-Sabotage (WLS).

<table>
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<tr>
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<th>Work</th>
<th>LaZe</th>
<th>Sabotage</th>
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Work, LaZe or Sabotage

a) What are all the Nash equilibria in the one-shot simultaneous-move game WLS?

b) Consider the twice-repeated game WLS(T=2, δ), where δ ∈ (0, 1] is the factor by which future utility is discounted, so that total utility for each player equals (payoff in period 1) + δ(payoff in period 2). In repeated game, a player can state what they will do in the second period based upon what the other player chooses in the first period. For what values of δ does there exist a Nash equilibrium (not necessarily subgame-perfect) in which each player Works for sure in period 1.

c) What are all the subgame-perfect Nash equilibria in WLS(T=2, δ)?

d) Consider the infinitely repeated game WLS(T=∞, δ). For what values of δ does there exist a subgame-perfect Nash equilibrium in which each player Works for sure in all periods.
6. True, False, Uncertain (Explain)

1. Increasing the productivity of all workers in an industry will increase employment.

2. Profits of gasoline stations fall when crude oil prices rise.

3. Health care costs are a burden to the firms providing health insurance to their workers.

4. If new technology caused cars to become more fuel efficient at no added cost to making a car, then the total demand for gasoline will fall.

5. The only supply curve with a constant price elasticity of 1 is a supply curve that is a 45 degree line, starting at the origin.

6. If unions raise labor productivity by 10% and raise wages by 10%, employment will not be affected.
1. Consider a model economy where the production of the homogeneous good depends on physical capital and human capital. That is,

\[ y_t = A k_t^\alpha h_t^{1-\alpha} \text{ where } 0 < \alpha < 1. \]

\( y_t \) is output per capita, \( k_t \) and \( h_t \) are the amount of physical and human capital used in production. Assume that each household accumulates both physical and human capital. As usual, the individuals maximize utility:

\[ \int_0^\infty \frac{c_t^{1-\theta} - 1}{1 - \theta} e^{-\rho t} dt \]

subject to

\[ y_t = c_t + i_{kt} + i_{ht} \]

where the investment of the two types of capital take on the following form:

\[ \dot{k} = i_k - \delta k \]
\[ \dot{h} = i_h - \delta h \]

\( \delta \) is the depreciation rate which is same for the two types of capital. Assume for simplicity that there is no population growth.

a) Set up the Hamiltonian and obtain the first order and transversality conditions of the individual’s maximization problem.

b) Does this economy exhibit endogenous growth? What is the rationale behind the existence or non-existence of endogenous growth? Explain using the expression for the growth rate.

c) Solve for the consumption along the optimal balanced growth path.

d) What is the necessary condition for utility to be bounded in this model?

2. Assume that the infinitely lived representative individual maximizes

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - \theta \log L_t \right] \right] \]
where $\beta$ is the discount factor, $L_t$ is the labor effort and $C_t$ is consumption. The individual supplies labor to the firms in return for the wage rate $w_t$, accumulates capital and rents it out to the firms at rate $1 + r^b_t$. Moreover, the individual holds government bonds $B_t$, which has a net rate of return $r^b_t$. While the government does not collect taxes from the return to bond holdings, they collect taxes from the return to capital. The tax rate is $\tau$. Hence, the individual's budget constraint is given by the following:

$$C_t + K_{t+1} + B_{t+1} = w_t L_t + (1 + r^b_t)B_t + [1 + (1 - \tau)r^b_t]K_t.$$ 

The production function of the perfectly competitive firm is:

$$Y_t = (AL_t)^aK_t^{1-a}$$

where $A$ is the constant level of technology. Assume for simplicity that there is no depreciation, and the firm maximizes profits: $\Pi_t = Y_t - (1 + r^b_t)K_t - w_t L_t$. Further assume that there is no growth in the steady state, therefore, technology, consumption, capital, bond holdings and output are constant in the steady state, and $A = 1$ in the steady state. Government purchases consumption goods, $G_t$, using the following rule:

$$G_t = G\hat{z}_t,$$

where $G$ is the steady state government spending. $\hat{z}_t$ is a government spending shock that follows an AR(1) process: $\hat{z}_t = \phi \hat{z}_{t-1} + \epsilon_t$, $E_{t-1}(\epsilon_t) = 0$, and is equal to 1 in the steady state. Finally, the government runs a balanced budget in every period satisfying:

$$B_{t+1} = (1 + r^b_t)B_t + G_t - \tau r^b_tK_t$$

a) Find the Euler equation for consumption and interpret it. Make sure to show all the first order conditions.
b) Obtain the first order conditions of the firm’s problem.
c) Find the steady state ratios for $\frac{L}{K}$, $\frac{Y}{K}$, $\frac{B}{K}$ and $\frac{G}{K}$ in terms of the parameters of the model.
d) What is the impact of $\tau$ on the steady state allocation of the two types of capital?
e) The solution to this model can be obtained by solving the system of linear expectational equations in the endogenous variables and the exogenous variable, $z$. Obtain the system of equations.
f) What is the necessary condition for the determinacy of this model? (Just write down the condition that applies to this model. You do not need to check if the condition holds.)

3. An unemployed worker receives each period a wage offer $w$ drawn from the distribution $F(w)$. The worker has to choose whether to accept the job - and therefore to work forever - or to search for another offer and collect $c$ in unemployment compensation. If the worker decides to accept the job, she must choose the number of hours to work in each period. The worker chooses a strategy to maximize

$$E \sum_{t=0}^{\infty} \beta^t u(y_t, l_t) \quad \text{where } 0 < \beta < 1,$$
and \( y_t = c \) if the worker is unemployed, and \( y_t = w_t (1 - l_t) \) if the worker is employed and works \((1 - l_t)\) hours; \( l_t \) is leisure with \( 0 \leq l_t \leq 1 \). Further, assume that \( F(0) = 0 \), \( F(\bar{w}) = 1 \) for some \( \bar{w} < \infty \).

(a) Analyze the worker’s problem. Establish that the optimal strategy is to choose a reservation wage. Show that the number of hours worked is the same in every period.

(b) Assume the same framework as in the previous literal but now the worker gets utility based on \( u(y_t, l_t) = y_t \). Establish the optimal strategy by the worker. Display the number of worked hours.

(c) Maintain the assumptions in part (b). Let \( v(w) \) be the optimal value function for an unemployed worker who has an offer \( w \) in hand. For simplicity assume \( c = 0 \). Show that Bellman equation \( v(w) \) associated to the problem has a unique equilibrium. (Hint: Consider the space of bounded continuous functions \( C[0, \bar{w}] \), the metric \( d_\infty(x, y) = \sup_{0 \leq w \leq \bar{w}} |x(w) - y(w)| \), and that the metric space \( C[0, \bar{w}], d_\infty) \) is complete).

4. Consider augmenting a neoclassical model with a transaction cost, \( s \), that is increasing in consumption, \( c \), and decreasing in real money holdings, \( m \). The economy is populated by a continuum of households in the unit interval. Each household orders her consumption and labor effort, \( h_t \), choices according with the utility function

\[
\sum_{t=0}^{\infty} \beta^t U(c_t, h_t).
\]

Here \( \beta \in (0, 1) \) is the subjective discount factor. It is assumed the \( U \) satisfies the usual Inada conditions. Holding real money balances is valuable because it facilitates consumption purchases. In particular, consumption purchases are subject to a proportional transaction cost that is proportional to consumption-based money velocity,

\[
\vartheta_t = \frac{P_t c_t}{M_t} = \frac{c_t}{m_t},
\]

where \( P_t \) denotes the nominal price level of consumption and \( M_t \) is nominal money. The transaction function \( s(\vartheta) \) satisfies: a) \( s \) is non-negative and twice differentiable; b) there exists a level of velocity \( \vartheta > 0 \) such that \( s(\vartheta) = s'(\vartheta) = 0 \); c) \( (\vartheta - \vartheta) s'(\vartheta) > 0 \) for \( \vartheta \neq \bar{\vartheta} \); and d) \( 2s'(\vartheta) + \vartheta s''(\vartheta) > 0 \) for all \( \vartheta > \bar{\vartheta} \).

Households have access to one-period nominal bonds, \( B_t \), which pay a gross nominal interest rate \( R_t \). The labor market is competitive with real wages \( w_t \). Firms transfer all their profits, \( \Pi_t \), to households. Hence, the households budget constraint is:

\[
P_t c_t + \left[ 1 + s(\vartheta_t) \right] + P_t \tau_t + M_t + B_t = M_{t-1} + R_{t-1} B_{t-1} + P_t \left[ w_t h_t + \Pi_t \right],
\]

where \( \tau_t \) denotes real taxes paid in period \( t \).

Final goods are produced by competing firms using the technology \( F(h_t) \). This technology is assumed to be strictly increasing and concave. Finally, the government
prints money, issues nominal one-period debt, and levies taxes to finance and exogenous stream of public consumption, denoted \( g_t \) and interest obligations on the outstanding public debt. Consequently, the government’s budget constraint is

\[ B_t + M_t + P_t \tau_t = R_{t-1} B_{t-1} + M_{t-1} + P_t g_t \]

(a) Derive the first-order conditions associated with the households’ optimal choices.

(b) Show that these conditions imply the following implied demand function for money

\[ \vartheta_t^2 s' (\vartheta_t) = \frac{R_t - 1}{R_t} . \]

(c) Show that the implied money demand is decreasing in the interest rate \( R_t \). What happens with the money demand function as \( R_t \) approaches 1? Hint: Use the assumptions imposed on \( s(\vartheta) \).

(d) Assume that government debt and purchases are nil at all times including at time 0 (maintain this assumption for the rest of the question). Under this assumption, what does the government do with its seignorage income?

(e) Carefully define a competitive equilibrium. Recall that the nominal interest rate can be below 1.

(f) In this setup, the Friedman rule establishes that \( R_t = 1 \) for all \( t \). Show that the Friedman rule can be supported as a competitive equilibrium outcome. HINT: Recall that the Friedman rule in steady state calls for deflation at the discount rate. Impose this condition and solve for the competitive equilibrium.

(g) Show that the Friedman rule is Ramsey optimal. To do so, proceed as follows: 1) Find the solution to the social planner’s problem:

\[ \max_{\{c, \vartheta, h\}} \sum_{t=0}^{\infty} \beta^t U (c_t, h_t) \]

subject to

\[ [1 + s(\vartheta_t)] c_t = F (h_t) \]

2) Show that the solution to this problem is identical to that from the Friedman rule (part f). Finally, argue that welfare under the social planner’s problem and that under the Friedman rule are the same. Use these results to prove the optimality of the Friedman rule.

(h) What is the level of taxes needed to finance the Friedman rule?
1. Production Theory Question

The Cobb-Douglas cost function is commonly represented as:

\[
\ln C(w, y) = \alpha_0 + \alpha_y \ln y + \sum_i \alpha_i \ln w_i \quad \text{or} \quad C(w, y) = e^{\alpha_0 y^{\alpha_y} \prod_i w_i^{\alpha_i}}
\]

where \( w_i \) is the price of input \( i \), \( y \) is total output, and the \( \alpha \) terms represent parameters of the cost function. For this cost function, answer the following questions:

(a) For the two input case \((i = 1, 2)\), solve for the factor demand equations.

(b) Solve for the marginal cost function.

(c) Solve for factor demand elasticities and the Allen-Uzawa elasticities of substitution for the two good case. (Note: The Allen-Uzawa elasticities of substitution are given by \( \sigma_{ij} = (CC_{ij})/(C_i C_j) \) where \( C = \) is total cost and \( C_i \) is \( \partial C/\partial w_i \) and so forth).

(d) To be consistent with theory, cost functions must satisfy a number of conditions. Outline the conditions (in terms of restrictions on the parameters) that are required to satisfy the following conditions:
   - Homogeneity (of degree one)
   - Symmetry of the elasticities of substitution
   - Constant returns to scale

(e) Inputs are characterized as substitutes or complements on the basis of the sign of the factor demand elasticities. Can an input be a complement for all other inputs in the Cobb-Douglas cost function?

(f) True, False, or Uncertain: In the general cost function case, symmetry of the elasticities of substitution implies symmetry of cross-price factor demand elasticities.
2. Household Production

Consider the labor/leisure decision of an individual. Initially, assume that the individual is self-employed and uses their labor $L$ to home-produce a good $Z$ using a production function $f(L)$, which they can sell to the market at a price of $P_Z$. They consume two goods—leisure ($H$) and a consumption good ($Q$) according to a utility function $U = U(H, Q)$. They must purchase the consumption good in the market at a price of $P_Q$ and face a time constraint of $H + L = T$, where $T$ is the total amount of time available.

As appropriate, answer the following questions and/or provide the requested information.

(a) Using a diagram, illustrate the labor allocation decision for the individual who has no outside work opportunities.

(b) Now suppose that the individual gets the opportunity to work outside the home at a wage of $w$. The individual supplies $M$ hours to the market, and the time constraint becomes $H + L + M = T$. Illustrate the time-allocation problem and identify the optimal solution.

(c) Now set up the optimization problem using the utility function and deriving the conditions for an optimal allocation of time. Be sure to explicitly identify any assumptions you make (hint—corner solutions). Assuming an interior solution, provide an intuitive discussion of the equilibrium conditions that will hold for an individual who works both outside the home and in the home.

(d) Now reconsider the graphical and analytical solutions. Suppose that the individual has the opportunity to hire outside labor at a wage of $w/2$ to work in their home. How would the answer change?

(e) Suppose the government imposes a minimum wage of $m$, such that $w > m > w/2$. Describe and illustrate the effect of this policy of the hours of hired help and the hours of work, $M$. Assume the price of $Z$ remains unchanged. Explicitly describe the income and substitution effects (if any) of this event.

(f) Now assume that the individual only has the option to work outside the home (i.e., disregard the home production part of the problem). In an effort to aid the poor, the government introduces a means-tested plan to support lower-income households. This plan gives a certain fixed amount of the consumption good $Q$ (call it $\bar{Q}$) to households whose labor incomes fall beneath a certain threshold. Using a diagram, explain how this affects the labor/leisure/consumption problem.
3. Risk and Uncertainty

Consider a standard Arrow-Debreu economy consisting of 2 identical consumers. Each agent has a wealth endowment of \( w \). The consumers are risk averse and have utility functions given by \( U(w) \). There also exists a single insurance company that is risk neutral. There are two possible states of nature: (1) No Loss and (2) Loss. In the case of a loss, each agent’s wealth decreases by \( L \). The probability of a loss is \( \pi \). The insurer offers a policy that will pay an indemnity payment of \( z \) units of wealth to each consumer in the case of a loss and charges consumers \( q \) per unit of \( z \) to provide this insurance.

(a) Set up the representative consumer’s optimization problem.

(b) If the insurance company is regulated by a central planner to ensure that it does not make profits, what is the premium amount per unit \( Z \) that the insurer should charge to ensure zero profits?

(c) Assuming that the insurer charges this premium amount, solve for the optimal level of insurance \( z \) that each consumer will choose to purchase. What is notable about this solution in terms of the amount of risk retained by each consumer?

(d) Now assume that the two consumers are no longer identical but instead have heterogeneous risks. The consumers are otherwise the same (same levels of wealth and same utility function). One consumer has a higher risk of loss \( \pi_H \) and the other has a lower risk of loss \( \pi_L \) (where \( \pi_H > \pi_L \)). Assume that the insurance company is unable to distinguish these different levels of risk and thus continues to view the two consumers as being identical. The insurer thus charges a common premium rate corresponding to average risks that falls somewhere between the two risk levels (\( \pi_L < q < \pi_H \)). Set up each consumers’ optimization problem and discuss the solution in the case where the company allows each consumer to select their own level of insurance. How do the relative levels of insurance compare (i.e., compare \( z_H \) and \( z_L \))?

(e) Now, consider a numerical example. Assume that you have been assigned to develop a crop weather insurance contract. This insurance is based on an index of total seasonal rainfall, given by \( I_t \). The index lies between 0 (the minimum possible amount of rain during a year) and 100 (the maximum possible amount of rain during a year). Optimal growing conditions occur at an index value of \( I_t = 50 \). Your company wants to offer a policy that will pay in the event of excessive dryness (which is defined by \( I_t < 25 \)) or excessive moisture (which is defined by \( I_t > 75 \)). To simplify the problem, assume that the coverage pays $1 for each unit the index falls beneath 25 or rises above 75. For example, in a year where \( I_t = 15 \) or \( I_t = 85 \), the contract would pay an indemnity of $10.

(f) Draw a picture that illustrates the risks you are trying to model (hint—draw a probability density function \( f(I) \) and identify the probability of loss).

(g) Derive an analytical expression for the probability of loss and expected level of loss under this contract (hint: express your answer in terms of the probability density function).

(h) Derive an analytical expression for the actuarially fair premium amount (i.e., the expected loss or expected payout) for this contract. What is the maximum amount that the insurer could pay out in any given year?

(i) Suppose that rainfall is uniformly distributed on the \([0,100] \) interval. Provide numerical solutions for the probability of loss and the actuarially fair premium amount.
4. Competitive Labor Markets

1. In a perfectly competitive workplace, all workers are interchangeable and have the same utility function:

   \[ U = U(Y, h) \], where \( U_r > 0 \), \( U_Y < 0 \), \( U_h < 0 \), \( U_{hh} < 0 \), and \( U_{hr} < 0 \)

where \( Y \) = daily income

\( h \) = daily hours of work.

Note that income, \( Y \), is a "good" and hours of work, \( h \), is a "bad".

Employers set work hours.

Workers can change jobs without cost and employers can change workers without cost. However, workers can only work in one job in a day (that is, they can not have two or more jobs, one after another, in any given day). In competitive equilibrium, when employers differ in working conditions, all workers get the same full wage (utility).

A. Assuming a competitive employer takes the market full wage (\( U^* \)) as given. How many hours of work would the employer require if they want the lowest average hourly cost (that is, the lowest value of \( Y/h \)). Illustrate the answer with indifference curve analysis, putting \( h \) on the horizontal axis. Show the result where the employer chooses 8 hours of work and a daily income (\( Y \)) of $80. [Hint: what slope does a ray from the origin have?] .

B. In a separate graph, draw the average hourly wage a firm must pay its workers for various hours of work (draw the general pattern), such that workers do not quit. Put the average hourly wage on the vertical axis and the hours of work on the horizontal axis.

C. Continuing with A, suppose the employer has a fixed daily cost per worker of \( F \). The employer now wants to minimize average hourly cost per worker (\( Y+F)/h \). Illustrate your answer in the indifference curve diagram in part A, assuming the worker is at the same utility level as in part A. [Hint: Put \( Y + F \) on the vertical axis and draw in the results from above, starting at \( F \).] Note: If you wish, you can draw a new diagram: just be sure to include the results from A. How does an increase in fixed cost affect the hours of work?

2. Holding hours and working conditions fixed, assume workers value a job based upon its hourly monetary wage (\( W \)) and its hourly level of fringe benefits (\( B \)):

   \[ U = U(W, B) \]

Letting each unit of \( B \) cost $1, the hourly cost of a worker to a firm equals \( W + B \).
A. Describe mathematically how a competitive firm will choose its wage-benefit mix, assuming it wants to attract workers at the lowest cost possible and that it must pay the market level of utility. Assume the labor market is perfectly competitive and workers all have the same utility function and are all interchangeable. The market is frictionless and all parties are well-informed.

B. Let \( U = W^{1/2} B^{1/2} \).

If the firm is spending $13 an hour on workers, solve for \( W \), \( B \), and \( U \). Next, assume a minimum wage of $9 is imposed on the firm. What is the worker’s new utility, holding costs constant? Draw the indifference curves illustrating these results. Put fringe benefits (B) on the horizontal axis. Label all relevant curves and intersections.

C. How much must the firm increase its total cost \( (W + B) \) by to give the worker the same utility before the minimum wage was imposed.

D. Draw the demand and supply curve diagram with labor \( (L) \) on the horizontal axis and full wages and full costs \( (W+B) \) on the vertical axis. The supply curve shows the quantity of workers who are willing to work at each full wage. The demand curve shows the full wage firms are offering at each level of \( L \). Initially, along the demand curve, \( MP = W + B = \text{Full Wage} \), where \( MP \) is the marginal revenue product of workers. While the minimum wage does not shift the \( MP \) curve, it does change the full wage firms are offering workers at each level of \( MP \). Utilizing the above model, draw the effect of the minimum wage on the labor market. Indicate how employment and full wages will be affected by the minimum wage.
5. Elasticity Questions

Show all work in the following problems. The work - not the results - is what will be graded. Use calculus to derive results and then covert results into elasticities.

Use the following terms:

Price Elasticity of Demand (absolute value): \( \eta = -\frac{\partial Q^D}{\partial P} \frac{P}{Q} \)

Price Elasticity of Supply: \( \varepsilon = \frac{\partial Q^S}{\partial P} \frac{P}{Q} \)

Partial Derivative: "\( \partial \)". Full Derivative: "\( d \)". For example, \( \frac{dQ^D}{Q} = \left[ \frac{\partial Q^D}{\partial P} \frac{P}{Q} \right] \frac{dP}{P} \).

1) A monopoly faces a constant elasticity of demand curve with an absolute price elasticity of \( \eta \). In the long run, it has a constant cost of output, so that ATC = MC at all Q. Show that its profit/price ratio \((P-ATC)/P\) equals \(1/\eta\). (hint: solve for the relationship between MR and P first).

2) Let the quantity demanded be a function of the market price, \( Q^D = Q^D(P) \), and the quantity supplied be a function of the market price, \( Q^S = Q^S(P) \). The market starts out in equilibrium and then the supply curve shifts horizontally left by S units, so that, at the new equilibrium, \( Q^P = Q^S - S \), where "\( Q^S \)" is the original supply curve before the shift. Derive the elasticity formula for how much the price will rise.

3) Let \( e_{ij} \) be the Marshallian elasticity of demand for good i with respect of price j. There are three goods. Letting only the price of good 3 change, derive how these three elasticities will be related \((e_{13}, e_{23}, \text{and } e_{33})\).

4) Let there be two goods (1 and 2) whose income elasticities of demand are \( \eta_{1Y} \) and \( \eta_{2Y} \), where Y is income. Prove that the share-weighted sum of income elasticities equals 1.
6. Versioning

Julia’s Restaurant has a monopoly on gourmet meals in her market. Each customer, on any given night, buys one meal. Julia, not the customers, determines the quality of each meal (q). Customers can only buy a meal or leave; they cannot buy more or less units of quality, q, than what is in the meal they buy. All customers are willing to pay more for a higher quality (q) meal. There are two types of customers, snobs and boors.

- Snob’s total value of a meal is $V_s(q)$ and his marginal value of q is $V_s’ > 0$.
- Boor’s total value of a meal is $V_b(q)$ and his marginal value of q is $V_b’ > 0$.
- A snob places a higher value on a meal than does a boor: $V_s > V_b$ for any given value of q.
- A snob places a higher marginal value on q than does a boor: $V_s’ > V_b’$ for any given value of q.
- $V'' < 0$ for snobs and boors.

The following diagram shows the plot of $V’$ and q. For example, at q=0, the snob’s $V’$ is c; at q=q_b, the snob’s $V'$ is zero. Area A is area between the $V'$ curves, from q=0 to q=q_1. Area B is the area between the $V'$ curves from q=q_1 to q=q_2. Area C is the area below the $V_b'$ curve from q=0 to q=q_1. Area D is the triangle under the $V_s'$ curve between q_1 and q_2. Area E is the triangle under the $V_s'$ curve to the right of q_2. If given only the choice of buying a meal with quality q_1, boors would be willing to pay up to area A (that is, $V_b(q_1)=A$) while snobs would be willing to pay up to area A+C.
The fraction of snobs in Julia's market is f and the fraction of boors is 1-f. There are N potential customers every night.

The marginal cost of producing q is zero.

1). If Julia sells meals with only one quality-level, describe how she would choose what price and quality to offer, assuming she cannot tell which customers are snobs. When will she sell to all and when will she sell only to snobs? Be explicit about the prices she can charge in this and all the following questions, using f and the areas from the diagram.

2). Julia can perfectly discriminate between boors and snobs. She can tell who is who. Will she offer only one quality level or two? What prices will she charge snobs and boors?

3). Now assume Julia cannot determine which customers are snobs or boors (this assumption applies to the rest of this question). Assume she decides to sell two qualities of meals, high quality meals ($q = q_3$) and low quality meals ($q = q_2$). What price will she charge boors for the low quality meal? If she offers the low quality meal for that price, and if snobs can buy it at that price, what is the highest price Julia can charge snobs for the high quality meal? How does this price compare to the price she charged snobs in part 2?

4) Suppose Julia reduces the quality of the low quality meal to $q_1$. Describe the following with the appropriate V terms and, in addition, using the areas in the diagram (use the four areas labeled with the letters A, B, C, and D):

The price she charges for the low quality meal.

The price she charges for the high quality meal.

5) Writing out the profit functions (using areas and f, the fraction of snobs), derive the values of f for which lowering the quality of the low quality meal from $q_2$ to $q_1$ is profitable to Julia.
Qualifying Exam: Macroeconomics
June 2010

Instructions. Do all questions.

1. Consider the following two aggregate production functions:

   \[ Y_t = AK_t^\alpha L_t^{1-\alpha} + BL_t \]
   \[ Y_t = AK_t^\alpha L_t^{1-\alpha} + CK_t \]

   where \( Y \) is aggregate output, \( K \) is the aggregate capital stock, \( L \) is aggregate labor, and \( A, B, \) and \( \alpha \) are constants satisfying \( A > 0, B > 0, C > 0, \) and \( \alpha \in (0,1) \). Labor \( L \) is constant (no population growth and no labor/leisure choice). Which, if either, of these production functions is capable of sustaining perpetual endogenous growth in income per worker, \( Y/L \), at a growth rate that does not go asymptotically to zero? Explain the property of each production function that leads to your answer and also explain the economic intuition for your answer. (For example, you might say for the first part of your answer, "Both functions can sustain perpetual endogenous growth because they use both capital and labor as inputs." You might say for the second part of your answer, "Because capital has a positive exponent in each function, households always have a positive marginal utility of capital and therefore always want to save an ever increasing fraction of their income.")

2. A consumer maximizes

   \[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \epsilon_t C_t - \frac{1}{2} C_t^2 \right] \right] \]

   subject to

   \[ A_{t+1} = R_t A_t + Y_t - C_t. \]

   The disturbance term \( \epsilon_t \) captures taste shocks, shocks to marginal utility. Assume for simplicity that the asset is a safe asset and that the interest rate is constant.

   (a) Set up the Bellman equation and derive the first order conditions.
   (b) Assume that \( \epsilon_t \) follows a first order autoregressive process (i.e. \( \epsilon_t = \gamma \epsilon_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \sigma^2) \) and \( 0 \leq \gamma \leq 1 \)). Characterize the consumption process and interpret the Euler equation.
   (c) What needs to be true about the parameter values for consumption to follow a random walk?

3. Suppose the atomistic firm has a production function \( F(K) \), pays \( G \) units of consumption for each new unit of capital \( K \) purchased, faces adjustment costs \( C(I) \) in installing capital with \( C' \) both positive and where \( I \) is investment, and faces a constant exogenous interest rate \( r \) and constant exogenous depreciation rate \( \delta \). The firm seeks to maximize the present value of the dividends it pays its owners:

   \[ \int_0^\infty \left[ F(K_t) - GI_t - C(I_t) \right] e^{-rt} dt \]

   where \( F(K_t) - GI_t - C(I_t) \) is the dividend (sales less costs) at time \( t \). The firm accumulates capital according to the usual equation:

   \[ dK_t/dt = I_t - \delta K_t \]

   and pays for investment with retained earnings (= sales revenue not distributed to shareholders).
(a) Use continuous time optimal control (the maximum principle) to prove formally that the firm chooses a path of capital that sets the marginal product of capital equal to the user cost of capital:

$$\frac{\partial F}{\partial K} = (r + \delta)(G + C') - d\psi/dt$$

where $\psi$ is the costate variable from the firm’s current-value Hamiltonian.

(b) Draw the phase diagram and use it to show that $K$ approaches its steady state value gradually.

(c) Suppose now that there are no adjustment costs: $C(I) = 0$ for all $I$. Derive formally the expression for the steady state value of $K$ and show that the firm will move $K$ to the steady state value instantly, not gradually.

(d) The standard Cass model has competitive firms with zero adjustment costs, $C(I) = 0$, so all firms must want to adjust their individual capital stocks instantly. Nevertheless, in the Cass model the aggregate capital stock does not move instantly to its steady state but instead adjusts gradually, approaching the steady state asymptotically. What is the economic reason that aggregate capital does not adjust instantly?

4. Assume that the infinitely lived representative individual maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ C_t^{1-\sigma} \left( \frac{M_t}{P_t} \right)^{1-\eta} - \frac{T_t^{1+\gamma}}{1+\gamma} \right]$$

where $\beta$ is the discount factor, $L_t$ is the labor effort and $C_t$ is consumption, $M_t$ is the money holdings, and $P_t$ is the aggregate price level. The individual supplies labor to the firms in return for the wage rate $W$, and holds a riskless bond, $B_t$, yielding a nominal return $R^b$. The bond is in zero supply in equilibrium. The individual’s budget constraint (in nominal terms) is given by the following:

$$P_t C_t + B_t + M_t = W_t L_t + R^b_{t-1} B_{t-1} + M_{t-1} - T_t.$$ 

Suppose that the monetary authority follows a constant money growth rule $M_t = \psi M_{t-1}$. Money is introduced into the economy through the lump-sum transfers, $T$, so that in equilibrium $T_t = M_t - M_{t-1}$. Firms operate a constant returns to scale production $Y_t = A_t L_t$, where $A_t$ is the technology process, and it follows an AR(1), in deviations from the steady state.

(a) Derive the household’s first order conditions.

(b) Obtain the steady state of the model.

(c) Show that the labor supply can be written in log-deviation form as $\bar{w}_t = a_1 \bar{C}_t + a_2 \bar{L}_t + \bar{\mu}_t$, where $w$ (small-case letter) denotes the real wage as a percentage deviation from its steady state value. What is the interpretation of the term $\bar{\mu}_t$ affecting this condition?

(d) Obtain the aggregate demand equation by log-linearizing the Euler equation, written in terms of output, $Y$. How does inflation affect aggregate demand?

(e) Note the necessary condition for the determinacy of this model.

5. Answer the following questions about the relation between income and consumption, providing enough explanation to show that you understand the economics at work in each case.

(a) Suppose we divide the labor force into two groups: those who work for themselves (self-employed) and those who work for somebody else (employees). It is a fact that the income of self-employed people is on average more variable from year to year than is the income of employees. On average, how would the two groups' marginal propensities to consume compare? Explain.
(b) Now discard the previous division of the labor force and instead divide the labor force into two new groups: those whose income is higher this year than last year (group 1) and those whose income is lower this year than last year (group 2). We then select two people, one from each group, who have the same income this year and look to see which person has the higher consumption. We then repeat this selection many times. Would the average of the consumption of the people from group 1 be higher, lower, or the same as the average of the consumption of people from group 2? Explain.

(c) We can collect cross-section data on households' income and consumption for a given year and regress households' current consumption on their current income. Typically when economists do that, they obtain a slope (i.e., coefficient on current income) that is less than one and a positive intercept on the vertical axis:

\[ C_i = a + bY_i \]

where \( C_i \) and \( Y_i \) are consumption and income of household \( i \), \( a > 0 \), and \( b \in (0, 1) \). In contrast, if we collect time series data for the aggregate economy and regress consumption on income, we will obtain a slope that is insignificantly different from one and an intercept of zero:

\[ C_t = Y_t \]

where \( C_t \) and \( Y_t \) are aggregate consumption and aggregate income in period \( t \). See the graph for a picture of the two regression lines. What is the explanation for this systematic difference between the consumption-income relations in these two different kinds of data?
1.) (35 min) True, false, or uncertain and explain. (Answer your choice of three of the four statements below. It’s your explanation that counts.):

a.) Trees cultivated for timber grow proportionately at a rate $g(t)$ that declines with time, $t$. Thus, $g'(t) < 0$. Assume the price per unit volume of timber is fixed. The optimal time to harvest is when $g(t)$ equals the interest rate.

b.) A risk-neutral manager knows for certain that a risk-averse employee will never shirk his/her responsibility to work hard. It therefore must be that the manager compensates the employee using a fixed wage rather than an incentive contract that ties the employee’s compensation to his/her output.

c.) In equilibrium, investments with greater expected return variances will have higher expected returns.

d.) If, holding all else the same, people discount utility of future consumption relative to present utility of consumption, it is still possible for equilibrium real risk-free interest rates to be negative.

2.) (45 min) Being an economic naturalist. Provide plausible economic answers to three of the four questions below.

a.) In the United States the traditional Thanksgiving Holiday meal is roast turkey. Indeed, the demand for turkey around Thanksgiving is substantially greater than at any other time of the year. But the retail price of turkey is lower around the Thanksgiving Holiday than at any other time of the year. Why is greater demand for turkey met with a lower price?

b.) Two key factors that affect mortality and life expectancy in some parts of Africa are AIDS, a sexually transmitted disease, and malaria, a disease spread mainly by mosquitoes. The incidences of AIDS and malaria are correlated. Why?

c.) When residential land prices go up, lots (building sites) typically decrease in size and increase in cost. That is, despite their smaller size, lots are more expensive than before. Builders then tend to construct fancier (and sometimes bigger) houses than before the increase in land prices. Why?

d.) Crop farmers in the Midwestern United States often plan corn on half their land and soybeans on the other half, even though corn almost always brings greater returns than soybeans do. Why don’t the farmers plant more corn and less soybeans?
3.) (15 min) All of the restaurants in town employ buffet pricing. That is, they sell “all you can eat” meals at a price that is uniform across customers. Customers are heterogeneous (some are teenage boys; some are grandmas), but the restaurants offer identical meals. Since the restaurants offer identical meals, competition forces them to all charge the same price; call this $p$. Now say that a new competitor (a new restaurant) opens but instead of employing buffet pricing, this restaurant employs cafeteria pricing. That is, the new restaurant charges customers for each item that they eat. The food is the same at the new restaurant, only the pricing arrangement is different. Will the entry of the new restaurant lead to a change in the price $p$ charged by the existing restaurants? If so, will $p$ increase or decrease? Explain.

4.) (30 min) The supply of widgets has a constant elasticity equal to $\mu$ and the demand for widgets has a constant elasticity equal to $-\theta$. Provide all your answers to the following questions with respect to these two parameters.

(i) If demand increases (shifts) exogenously by 5 percent, by how much does the quantity demanded of widgets change? By how much does the price of widgets change?

(ii) For the demand shift in (i), what is the change in consumers’ surplus? What is the change in producers’ surplus?

(iii) If widget sales are subjected to a tax of 10 percent on sellers, how much does this cause price (tax included) and quantity of widgets sold to change?

(iv) By how much does the tax in (iii) cause consumers’ and producers’ surplus to change? How would the answer differ if the tax were levied on consumers rather than producers?

5.) (25 min) Indirect profit functions are convex in prices. Indirect utility functions are also convex in prices. Thus, greater price variability unambiguously increases expected profits and expected utility. This implies that policies that invoke greater price variability are strongly welfare improving. Explain what is right, what is wrong and what is missing from this line of reasoning.

6.) (25 min) Consider a competitive model with one factor, two outputs, and two firms with production functions $x_1 = f_1(L_1) = \sqrt{2L_1}$ and $x_2 = f_2(L_2) = \sqrt{2L_2}$. Consumer $a$ is endowed with 25 units of the factor of production, owns no share in the firms, and has utility function $U_a(x_{1a}, x_{2a}) = x_{1a}x_{2a}$. Consumer $b$ owns both firms but has no endowment, and has utility function $U_b(x_{1b}, x_{2b}) = \sqrt{x_{1b} + x_{2b}}$. Find a competitive equilibrium in this model, including prices, production and allocations.
7.) (25 min) In period 1, Parent is considering allocating time between consumption $c$ and "guilt inducement," $g$. In period 2, Child chooses between fun $f$, and visiting Parent, $v$. Parent’s lifetime utility function is $U(c, g, v) = c - g^2 + 2v$, where $c + g \leq 1$. Child’s lifetime utility is $V(f, g, v) = (f)^{1/2} - g/v$, where $f + v \leq 1$. All variables are constrained to be non-negative. What is the outcome in terms of $c, g, f$ and $v$? Is this outcome efficient? Discuss.

8.) (35 min) Consider the concepts of an “incredible threat” and an “incredible promise.” An incredible threat is a situation where one person threatens to take an action if the second person takes an action that is unfavorable to the first person, but the second person does not believe the first person’s threat, and so second person takes that action anyway. An incredible promise is a situation where one person promises to take a particular action if a second person takes a particular action, but second person does not believe the first person and so does not take the action requested by the first person.

i) Provide economic examples of an incredible threat and an incredible promise.

ii) Considering the game That’s Incredible!, for what values of $a$, $b$, $c$ and $d$ would you say we have a situation where we have an incredible threat? For what values of $a$, $b$, $c$ and $d$ would you say we have a situation where we have an incredible promise?

iii) Does Nash equilibrium generally allow for incredible threats? Does Nash equilibrium generally allow for incredible promises? Does Nash equilibrium tell us anything about the credibility of threats and/or promises?

That’s Incredible!
1. Briefly explain each of the following and its significance.
   
   a) The First and Second Welfare Theorems

   b) The Weak Axiom of Revealed Preference

   c) Walras Law

   d) Perfect Bayesian Equilibrium

   e) The Core

2. You've hired a risk-neutral man to deliver TVs to customers who have bought them. His job is to go to the warehouse, locate the TV that you've sold to a customer, and then deliver it to the customer. You suspect that the deliveryman may steal the TV (and then sell it for M dollars). If he does this, he will claim that the TV wasn't in the warehouse in the first place. Both you and he know that if he does this, you will only catch him with probability $p$. That is, sometimes you won't know that he stole the TV. You've promised to pay the deliveryman a wage $w$, but if you catch him stealing the TV you won't pay him this wage (he gets no wage).

   a) Assume that if you catch the deliveryman stealing the TV, you cannot recover the TV (he gets to keep it). How much do you have to pay the deliveryman (what is $w$), so that you will feel confident that he won't steal the TV?

   b) Assume instead, that when you catch the deliveryman stealing the TV you not only don't pay his wage but you also recover the TV. Now, how much do you have to pay the deliveryman (what is $w$) so that you feel confident that he won't steal the TV?

   c) Say that you can recover the TV when you catch the deliveryman, but only at a cost of $C$ dollars. When should you recover the TV?
3. An individual's demand function for two goods, $x_1$ and $x_2$ are:

$$x_1 = a_1 + b_1 p_1 + b_{12} p_2$$
$$x_2 = a_2 + b_2 p_1 + b_{21} p_1$$

where $a_1$, $a_2$, $b_1$, $b_2$, $b_{12}$, and $b_{21}$ are fixed parameters and $p_1$ and $p_2$ are prices for the two goods.

a) What restrictions on the parameters does theory imply?

b) What is the associated money metric indirect utility function? 
   (Recall: the money metric indirect utility function gives the minimum expenditure at prices $\mathbf{p}$ necessary to purchase a bundle at least as good as that purchased at prices $\mathbf{q}$ with income $m$. Mathematically, it is:

$$\mu(\mathbf{p}, \mathbf{q}, m) = e(\mathbf{p} | v(\mathbf{q}, m))$$

where $e()$ is the expenditure function and $v()$ is the indirect utility function, and thus $\mu(\mathbf{p}, \mathbf{q}, m)$, when taken with respect to $\mathbf{q}$ and $m$, is just a monotonic transformation of the indirect utility function and thus itself an indirect utility function.)

4. Provide brief intuitive answers to each of the following.

a) Girl Scout cookies come in many varieties. Some are very popular (Thin Mints are the runaway best seller); others less so. Despite the differences in demand across varieties, they all sell for the same price. Presuming that the intent of the Girl Scouts is to raise money, wouldn't it be better to charge different prices for different varieties? Why don't the Girl Scouts do this?

b) Salesmen get paid, at least partly, by commission (a percentage of the dollar value of the goods that they sell), but production workers almost always get paid an hourly wage that is not affected by how much they produce. Why aren't production workers paid like salesmen? And why aren't salesmen paid like production workers? Among salesmen, it is common for pay to be part commission and part base pay (unrelated to sales). What determines the amount of base pay?

c) At the Grand Bazaar in Istanbul (the largest city in Turkey and a popular tourist destination both by bus (or car) and by cruise ship) prices are determined by haggling. At the Silk Market in Bursa (a smaller inland city in Turkey, somewhat less popular with tourists and accessible only by land), prices are also determined by haggling. However, the bargaining range (the difference between the highest prices paid and the lowest
is much larger at the Grand Bazaar than at the Silk Market. At the Grand Bazaar, bargaining really hard may get you a price that is 40% less than the initial offer; while at the Silk Market, hard bargaining may result in a price that is only about 5% less than the initial offer. Why is the bargaining range so different between the two markets?

5. On nice summer Sundays, Calvin likes to set up a stand in the local park and sell cups of lemonade for extra money. Lemonade costs Calvin $c$ per cup to produce. His mother makes the lemonade for him, and he enjoys his time at the park anyway, so he has no opportunity costs except for the lemonade. His decisions are (i) how much lemonade to buy and bring with him to the park to sell; and (ii) the price to sell his lemonade. Suppose Calvin believes there is a 50 percent chance it will be a high-demand day, in which case the lemonade demand schedule is: \( Q = 2a - bP + \varepsilon \), and a 50 percent chance it will be low-demand day, in which case \( Q = a - bP + \varepsilon \), where \( Q \) is quantity demanded, \( P \) is price, and \( \varepsilon \) is a uniformly distributed random variable on the interval \((-\alpha, \alpha)\). (Note: the parameter \( a \) is also in the demand function). Suppose that as soon as Calvin gets to the park, he knows whether it is a high-demand day or a low-demand day and can set his price accordingly, but he does not observe \( \varepsilon \) and it's too late to change the quantity of lemonade he brought with him. For simplicity, assume he cannot change his price as the day progresses.

a) Taking as given the amount of lemonade Calvin brought to the park, what price will he choose if it's a low demand day? If it's a high demand day?

b) How much lemonade will Calvin bring to the park?

c) What is the probability Calvin will sell all the lemonade he brings to the park?

d) Suppose that Hobbs could get up early, go to the park, and call Calvin to let him know whether it's a low-demand day or high-demand day before he decides how much lemonade to bring with him. What is the most Calvin would be willing to pay Hobbs to go to the park early? How would he change lemonade prices?
Qualifying Exam: Macroeconomics
January 2011

Instructions. Do all questions.

1. Robinson Crusoe lives on an island and has no contact with other people. Suppose Crusoe's utility at time $t$ is a strictly concave function $u$ of consumption $c_t$ and labor $n_t$:

$$u_t = u(c_t, n_t)$$

where $u_c > 0$, $u_{cc} < 0$, $u_n < 0$, and $u_{nn} < 0$. Crusoe produces goods $y_t$ with his labor:

$$y_t = f(n_t)$$

$$= A_t n_t^\alpha + B_t$$

where $A > 0$, $B > 0$; and $1 > \alpha > 0$.

(a) Assume first that Crusoe has no contact with any other people. Derive formally the solution for Crusoe's optimal time paths of consumption and labor. Describe the relation between $c_t$ and forecastable changes in future values of $B_{t+i}$.

(b) Suppose now that Crusoe discovers a friendly neighboring tribe with whom he can engage in borrowing and lending. Crusoe is an atomistic agent in this larger economy. Derive again Crusoe's optimal time paths of consumption and labor, subject to the usual condition that lifetime spending must equal lifetime production. Also, describe the relation between $c_t$ and forecastable changes in future values of $B_{t+i}$.

2. In the fall of 1991, it was obvious that the United States was going to engage in a short war with Iraq sometime in the first half of 1992. In fact, the war did happen and was over by the spring of 1992. Assume population and labor supply are fixed, utility is a concave function $u$ of consumption per person $c$ only, $u(c)$, and that output $y$ is given by $y = f(k)$, where $f$ is concave and $k$ is the capital/labor ratio. Assume also that the US started in steady state at the moment that people became aware of the coming war.

Use the Cass growth model (the standard optimal control solution for the centrally planned economy, with no technical progress) to explain the time path of real output $y$ in the US from the fall of 1991 to the end of 1992.

3. The Solow model treats population growth as exogenous. Suppose instead that the percentage population growth rate is proportional to the difference between income per capita $y$ and some minimal subsistence level of income per capita $m$; that is,

$$\left(\frac{dL}{dt}\right) \left(\frac{1}{L}\right) = p(y - m)$$

where $L$ is population and $p$ is a constant.

(a) Set up the Solow-type growth model with this modification and derive the equation for the growth of the capital/labor ratio.
(b) Draw the usual graph, with appropriate modifications, to show the steady states in the model and discuss their stability.

4. A consumer maximizes

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \log C_t \right] \]

subject to the following wealth accumulation equation

\[ Z_{t+1} = (1 + r_t)(Z_t - C_t) + Y_{t+1} \]

where \( Z \) is the beginning-of-period wealth, and \( Y \) is the stochastic endowment income. In addition to the wealth accumulation equation, the individual faces a liquidity constraint preventing the individual from borrowing against future income. Hence the individual's assets, \( A_t = Z_t - C_t \) cannot be negative at any point in time.

a. Set up the Bellman equation and derive the first order conditions.

b. Characterize the consumption process and interpret the Euler equation.

c. Does the permanent income hypothesis hold in this case? Explain.

5. Assume that the infinitely lived representative individual maximizes

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t (1 - L_t)^{\chi}}{1 - \gamma} - 1 \right] \]

where \( \beta \) is the discount factor and \( \gamma \) is the coefficient of risk-aversion. The total amount of time available to agents in each period is normalized to 1. Thus, \( 1 - L_t \) is the leisure in period \( t \), and \( \chi \) is the inverse of the elasticity of labor supply. The individual supplies labor to the firms in return for the wage rate \( w \), accumulates capital and rents it out to the firms at rate \( R \). Hence, the individual's budget constraint is given by the following:

\[ C_t + K_{t+1} = R_t K_t + w_t L_t \]

The production function of the perfectly competitive firm is:

\[ Y_t = (A_t L_t)^{\alpha} K_t^{1-\alpha} \]

where \( A_t \) is the productivity shock. Assume for simplicity that there is no depreciation, and the firm maximizes profits:

\[ \Pi_t = Y_t - R_t K_t - w_t L_t. \]

There is no growth in the steady state, therefore, technology, consumption, capital and output are constant in the steady state. Assume that \( A = 1 \) in the steady state.

a. Find the Euler equation for consumption and interpret it.

b. How does the elasticity of labor supply affect the optimal consumption-leisure choice?
c. Find the steady state ratios for $\frac{L}{K}$, $\frac{Y}{K}$ and $\frac{G}{K}$ in terms of the parameters of the model.

d. The solution to this model can be obtained by solving the system of linear expectational equations in $C, K, L$ and $A$. Obtain the system of equations. In obtaining the system, use the following assumption:

- The percentage deviations of $A_t$ from its steady state value follow an AR(1) process: $\hat{A}_t = \phi \hat{A}_{t-1} + \epsilon_t$, $E_{t-1}(\epsilon_t) = 0$
1. A competitive firm produces output $y$ using inputs $x_1$ and $x_2$ according to the following production function:

$$y = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$$

(a) What is the cost function for the firm?
(b) Derive the factor demands for $x_1$ and $x_2$ conditional on the amount produced. Are $x_1$ and $x_2$ substitutes? Explain.
(c) Derive the factor demands for $x_1$ and $x_2$ conditional on output price. Are $x_1$ and $x_2$ substitutes? Explain.

2. The mineral Zirconium is sold in a competitive market, with an initial equilibrium quantity sold of 100 tons at a price of $40/ton. The government decides to raise revenue by taxing Zirconium sales. They charge purchasers of Zirconium a tax of 10% of the equilibrium price received by sellers. The elasticity of demand for Zirconium is -0.5 and the elasticity of supply is 2.0.

(a) What is the equilibrium price received by suppliers after the tax is imposed? What is the equilibrium price paid by consumers, including the tax?
(b) Calculate a measure of the welfare change to consumers that results from the imposition of the tax.

3. Consider two proposals to increase the incomes of tobacco farmers. The first is a Direct Payment plan, in which the government guarantees to producers a price per pound of $P^*$, which is above the laissez-faire equilibrium price. Under the Direct Payment plan, the government would pay to farmers any difference between $P^*$ and the market price; this supplement would be paid for each pound of tobacco that a farmer sells on the market. The second plan is a Government Purchase plan. The government would buy on the market the amount of tobacco necessary to force the price to $P^*$. The tobacco purchased by the government would be dumped into the ocean.

(a) Which of the two programs would be preferred by tobacco consumers? Which would be preferred by the American Cancer Society? Explain.
(b) Which of the two plans would cost the government the most to operate? Your answer should include the elasticity of demand for tobacco.
4. Consider a competitive market for a good. Aggregate inverse demand for the good is \( P = \alpha Q^{-\beta} \) where \( Q \) is the total amount bought at price \( P \) and \( \alpha \) and \( \beta \) are positive parameters. An unlimited number of potentially producing firms each has a cost function \( c = f + \mu q^\gamma \), where \( c \) is the total cost of production, \( f \) is fixed cost, \( q \) is the quantity produced by a single firm, and \( \mu \) and \( \gamma \) are parameters. A firm incurs the fixed cost if it produces positive output but not if \( q = 0 \). Assume \( \gamma > 1 \).

(a) Derive expressions showing the number of producing firms \((n)\), price \((P)\), quantity produced by each firm \((q)\), and total quantity produced \((Q)\) as a function of the parameters.

(b) What happens in the long run to the number of producing firms \((n)\), price \((P)\), quantity produced by each producing firm \((q)\), and total quantity produced \((Q)\) if fixed cost (the parameter \(f\)) increases?

5. Consider a market for a good that has a dominant firm with a constant marginal cost of production equal to \( z \) and fixed cost equal to \( f \). The dominant firm is not necessarily a monopolist; it must compete with a competitive fringe that has a perfectly elastic supply curve at \( P = P^* \). Aggregate inverse demand for the good is given by \( P = \alpha - \beta Q \) where \( Q \) is the total amount bought at price \( P \) and \( \alpha \) and \( \beta \) are positive parameters. Assume \( z < P^* < \alpha \).

(a) What is the equilibrium price, quantity produced by the dominant firm and quantity produced by the competitive fringe? (Hint: There are multiple characterizations of the solution depending on the values of the parameters.)

(b) Suppose fixed cost increases for both the dominant firm and each competitive-fringe firm. Describe what happens in the long run to price, quantity produced by the dominant firm and quantity produced by the competitive fringe. Make sure you characterize each possibility depending on the values of the parameters. A graph or two may help.
6. (a) Write down your own two-player Prisoners Dilemma. Use different numerical payoffs than you recall seeing in ECG 702, a textbook or some other venue.

(b) For the game you develop, consider the twice-repeated version of the game with no discounting of payoffs in the second period. (This would be PD(T=2, δ=1) using the notation from ECG 702.) What are the subgame-perfect Nash equilibria to this game?

(c) Write down the normal form of the twice-repeated game.

(d) Find all outcomes that are rationalizable in the twice-repeated game.

(e) Find all outcomes that survive iterated deletion of weakly dominated strategies in the twice-repeated game.

(f) Find all outcomes that are Nash equilibria in the twice-repeated game.

(g) For an infinitely-repeated version of the game, fully characterize a pair of strategies that would support the cooperative outcome in all periods of the game as a subgame-perfect Nash equilibrium for some discount factor δ < 1, where δ is the proportional factor by which period t + 1’s payoff is valued relative to period t.
Qualifying Exam: Macroeconomics  
June 1, 2011

Instructions: You have four hours to complete all the following questions. Point allocations are denoted in the questions. Please allocate your time wisely (browse through the test, allot your time, and attempt to answer all questions).

1. (20 points) Below are year-to-year percentage growth rates of real GNP, total consumption, consumption of durable goods, and consumption of non-durable goods for the United States during 1979-1984. As you can see, Y swings more from year to year than C does, C durable on average swings more than C, and C non-durable on average swings less than C. The term usually used to describe this situation is to say that C durable is the most volatile variable, Y is the second-most volatile, C is third, and C non-durable is the least volatile. How would you explain the relative volatilities of these four variables?

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<td>%ΔY</td>
<td>+6.0</td>
<td>-0.5</td>
<td>+2.5</td>
<td>-2.1</td>
<td>+3.7</td>
<td>+6.8</td>
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<td>%ΔC</td>
<td>+3.3</td>
<td>+0.1</td>
<td>+2.0</td>
<td>+1.3</td>
<td>+4.8</td>
<td>+5.3</td>
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<tr>
<td>%ΔC durable</td>
<td>0.0</td>
<td>-6.2</td>
<td>+2.5</td>
<td>-0.3</td>
<td>+12.1</td>
<td>+13.0</td>
</tr>
<tr>
<td>%ΔC non-durable</td>
<td>+4.0</td>
<td>+1.3</td>
<td>+1.9</td>
<td>+1.6</td>
<td>+3.5</td>
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2. (20 points) Consider the simple New Keynesian model with Calvo pricing, variable labor supply, fixed capital stock, an increasing, concave production technology, and CES preferences over consumption and leisure. The consumption Euler equation implies

\[ x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r^n_t) \]  

(1)

where all variables are in terms of log deviations from steady state, \( \pi \) is the inflation rate, \( i \) is the nominal interest rate, \( r^n \) is an exogenous i.i.d. demand shock, and \( x \) is the output gap between output in this economy and output in an economy without monopolistic competition. The Phillips curve is

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]  

(2)

The optimal monetary policy in this economy ("optimal" in the sense it would give private agents the highest welfare) would be one that implies

\[ x_t = 0, \ \ \pi_t = 0 \]  

(3)
(a) (5 points) If the monetary authority adopts the rule \( i_t = r_t^n \), can it ensure the equilibrium with \( x_t = \pi_t = 0 \), for all \( t \), will occur? Prove your answer.

(b) (5 points) One can show that the optimal outcome in (??) is achieved when the monetary authority follows the strict inflation targeting rule \( \pi_t = 0 \). Is this rule consistent with the Friedman rule? Explain.

(c) (5 points) Suppose the central bank's policy instrument is the nominal interest rate. How does the central bank set the nominal interest rate in equilibrium, under the strict inflation targeting policy in part (b)?

(d) (5 points) Consider a monetary policy that targets the price level:

\[
I_t = \varphi(P_t)
\]

where \( I_t \) is the level of the nominal interest rate and \( P_t \) is the price level. Let \( p_t \) denote log deviations of the price level from its steady state \( P \) (so that \( \pi_t = p_t - p_{t-1} \)). Log-linearize the monetary policy rule and derive the solutions for equilibrium allocations of \( x_t, p_t, \) and \( i_t \) assuming \( \frac{\varphi'(P)P}{\varphi'(P)} > 0 \) and \( \frac{\varphi''(P)P}{\varphi'(P)} > 0 \).

3. (20 points) Suppose we have labor-augmenting technical progress, so that the production function has the form

\[
Y_t = F(K_t, A_t L_t)
\]

where

\[
A_t = A_0 e^{st} \quad (6)
\]

\[
L_t = L_0 e^{nt} \quad (7)
\]

The rate of technical progress \( g \) is a positive constant. The rate of population growth \( n \) equals the birth rate \( b \) minus the death rate \( d \). We suppose \( F \) is homogeneous of degree 1 in \( K \) and \( AL \) (that is, there is constant returns to scale). Use the Solow-Swan model to answer the following questions.

(a) (8 points) Suppose this economy is Western Europe in 1346 AD, when the Black Death struck. The Black Death was a plague that killed about one-third of Europe’s population. Describe the time path of output per person (\( Y/L \)) in the 50 years before the Black Death and the 50 years after it; explain why it has the shape it does. To simplify the discussion, treat the Black Death as happening in an instant rather than being spread over many years and treat the birth rate \( b \)
and the death rate \( d \) as constants with \( b > d \). That is the way many diseases, such as influenzas, actually behave.

(b) (8 points) Suppose now that instead of being a one-time event, the Black Death was recurrent, which is the way it actually behaved. For simplicity, assume that the Black Death was a permanent and constant phenomenon. (In reality, it came in waves every decade or two, but that’s unnecessarily difficult to model.) Modify the original model as necessary to capture this behavior of the Black Death and describe the time path of \( Y/L \).

(c) (4 points) A final fact about the Black Death is that it gradually became less and less virulent and virtually died out in Europe sometime in the 18th century. Elsewhere, outbreaks continued into the 20th century. (12 million people in India alone died from the Black Death over the period 1896-1930.) Modify your answer to the previous part to allow for the gradual disappearance of the Black Death, and describe the time path of \( Y/L \).

4. (20 points) Consider an economy with a continuum of identical individuals on the unit interval who live forever. There is no population growth. Individual \( i \in [0, 1] \) maximizes utility given by

\[
\sum_{t=0}^{\infty} \beta^t u(c_t^i), \quad 0 < \beta < 1
\]

where \( c_t^i \) is consumption at time \( t \) and \( u \) has standard properties. An individual \( i \)'s flow budget constraint is given by

\[
c_t^i + K_{1t+1}^i + K_{2t+1}^i + b_{t+1}^i = F(K_{1t}^i, K_{2t}^i) + (1 - \delta)K_{1t}^i + (1 - \delta)K_{2t}^i + (1 + r_t)b_t^i
\]

where \( K_{jt}^i, j = 1, 2 \) is capital of type \( j \). The production function has standard properties including homogeneity of degree one. \( b_{t+1}^i \) denotes one period risk-free private bonds that pay a net return of \( r_{t+1} \) in period \( t + 1 \). \( K_{jt}^{i0}, b_0^i \) are the initial endowments. The \( ith \) agent’s lifetime budget constraint is given by

\[
\sum_{t=0}^{\infty} \frac{c_t^i + K_{1t+1}^i + K_{2t+1}^i}{\prod_{s=0}^{\infty}(1 + r_s)} = \frac{\sum_{t=0}^{\infty} F(K_{1t}^i, K_{2t}^i) + (1 - \delta)K_{1t}^i + (1 - \delta)K_{2t}^i}{\prod_{s=0}^{\infty}(1 + r_s)}
\]

(a) (4 points) Write down, as precisely as possible, Bellman’s equation for individual \( i \)'s problem.

(b) (4 points) Let \( v_0^i(K_1^i, K_2^i, b^i) = 0 \) be an initial guess for the value function for the
individual $i$'s problem. Explain how this guess $v_i^t$ can be used to find the value function. Note: You do NOT have to solve for the value function.

(c) (4 points) Specify all the assumptions necessary to apply the Contraction Mapping Theorem. What does this theorem ensure?

(d) (4 points) Write down, as precisely as possible, Bellman's equation for the representative agent's problem. How do the allocations of the individual household in market equilibrium relate to the allocations of the representative household? Explain how the market achieves that relation.

(e) (4 points) Derive and interpret the Euler equations for the representative agent's problem. State any transversality conditions for the problem.

5. (20 points) It often is said that an increase in government purchases stimulates the economy - that is, raises employment and output.

Suppose the economy starts in steady state when government purchases $G$ unexpectedly increase. Suppose the increase is permanent and once it happens is always expected to be so. Finally, suppose the government always finances its purchases with strictly proportional income taxes levied on wage and rental income (equivalently, on gross production) where tax revenue $T$ equals the tax rate $\tau$ times output $Y$: $T = \tau Y$. The government adjusts $\tau$ as necessary to balance the government's budget (no exemptions or deductions, no government bonds, no printing money). That is, the government sets purchases and then raises whatever taxes are needed to pay for the purchases. In this economy, identical households provide labor $L$ and buy consumption $C$ and investment goods $I$. Identical firms produce output $Y = F(K, L)$, which is divided among consumption, investment, and government goods. Firms rent capital and labor to carry out production. The households own the firms. Capital depreciates at the constant proportional rate $0 < \delta < 1$ and so changes as $\frac{dK}{dt} = I - \delta K$.

(a) (10 points) Use the representative agent (central planning) approach and the continuous-time maximum principle (optimal control) to derive formally the time paths of output and employment, starting from shortly before the increase in government purchases and going to the new steady state. Draw a graph showing the paths of output and employment. Show your derivation.

(b) (6 points) Does the increase in government purchases ever stimulate output? Never stimulate it? Stimulate it part of the time? Explain the economics underlying your answer.
(c) (4 points) What can you say about the change in the households lifetime utility, given by $\int_0^\infty U(C_t, L_t)e^{-\rho t} dt$?

[Some interesting exercises to try in your spare time are to consider a temporary increase in government purchases that is understood to be temporary when it happens (such as a war), an increase in the tax rate accompanied by whatever change in purchases balances the budget, and the case where labor is fixed rather than variable. Working out all those cases will give you a pretty complete understanding of how fiscal policy works. Then, if you want to ruin your day, you can compare your results to the discussions by politicians and the news media.]
1. Berna's utility function over two goods is given as:

\[ U(q_1, q_2) = \min\{\frac{q_1}{2}, q_2\}. \]

Berna's income is 100 dollars. She faces prices of \( p_1 = 3 \) dollars and \( p_2 = 4 \) dollars. Calculate the *compensating variation* for Berna if she were to face an increase in prices to \( p_1 = 5 \) and \( p_2 = 10 \). Calculate the *equivalent variation* of the same change in prices.

2. The country of Euphonia produces trombones in two regions, Highland and Lowland. In Lowland there are 100 identical trombone producers, each of which has the following marginal cost curve:

\[ m_{CL} = 2 + q_L, \]

where \( q_L \) is the quantity produced by an individual Lowland producer.

In Highland there are 100 identical trombone producers, each of which has the following marginal cost curve:

\[ m_{CH} = 3 + q_H, \]

where \( q_H \) is the quantity produced by an individual Highland producer.

Trombones are sold on the world market at a price of 6 dollars per unit. Variations in production in the two regions have no influence on the world price.

The government of Euphonia restricts the production and sale of trombones by requiring marketing certificates to sell trombones: each certificate entitles the owner to sell one trombone on the world market. The government issues 400 certificates, split between the two regions: 200 certificates are given to residents of Lowland and 200 certificates are given to residents of Highland. Certificates can be traded within each region, but cannot be traded between the two regions.

(a) What will be the price of a certificate in Lowland? What will be the price of a certificate in Highland?

(b) If it became legal to trade certificates between the two regions, what would be the equilibrium distribution of production between the two regions, and what would be the price of certificates?

(c) Who would be in favor of making certificates tradeable? Lowland producers? Highland producers? Lowland certificate owners? Highland certificate owners?
3. In a competitive market for widgets, the price elasticity of supply is $\epsilon$, the price elasticity of demand $-\eta$, and the elasticity of demand with respect to average consumer income is $\phi$. All parameters are positive.

(a) Derive an expression for the change in equilibrium price if average consumer income were to increase by 10%?

(b) Derive an expression for the change in producer welfare that results from the exogenous increase in income. Is the gain to producers larger if demand is more income elastic? Is the gain to producers larger if demand is more price elastic?

4. Consider the game Ro Sham Bodacious.

<table>
<thead>
<tr>
<th></th>
<th>Rock</th>
<th>Paper</th>
<th>Scissors</th>
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<tbody>
<tr>
<td>Rock</td>
<td>$a, a$</td>
<td>-1, 1</td>
<td>1, -1</td>
</tr>
<tr>
<td>Ted</td>
<td>1, -1</td>
<td>$b, b$</td>
<td>-1, 1</td>
</tr>
<tr>
<td>Paper</td>
<td>-1, 1</td>
<td>1, -1</td>
<td>$c, c$</td>
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<tr>
<td>Scissors</td>
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Note that $a, b$ and $c$ are variables in this problem, representing how much Bill and Ted like the outcomes when they both choose the same action. If $a = b = c = 0$, then the game is the well-known hand game.

In parts (a) through (c) assume that $a = b = c = 0$.

(a) Which of Ted’s actions (i.e., pure strategies) are rationalizable?

(b) Which of Ted’s actions survive iterated deletion of weakly dominated strategies?

(c) What are all the Nash equilibria of this game (including mixed strategies)?

Now consider a twice-repeated version of Ro Sham Bodacious, and let $a, b$ and $c$ take on any value. Assume payoffs in the second period receive equal weight to those in the first period (i.e., the discount rate is zero)

(d) Find some values of $a, b$ and $c$ that support play of (Rock, Rock) in the first period as part of a subgame perfect Nash equilibrium, but for which (Rock, Rock) is not a Nash equilibrium in the single-shot version of the game. Carefully explain your reasoning.
5. Consider an exchange economy with two households with utility functions \( U_a(X_a, Y_a) = X_a + Y_a \) and \( U_b(X_b, Y_b) = (X_b Y_b)^{1/2} \). Agent \( a \) is endowed with 1 unit of \( X \) and agent \( b \) is endowed with 1 unit of \( Y \). (Note the difference between the two utility functions.)

(a) Find the Pareto efficient allocations in this economy.

(b) Find the core of this economy.

(c) Suppose there are two agent \( a \) types and two agent \( b \) types. Show that the core allocation that was worst for agent \( b \) in the single replicate economy (part (b) above), is not in the core of twice replicated economy.

(d) Suppose the economy is replicated many times, so that there are \( N \) of each type of agent, and \( N \) is large. What allocation will the core converge to? (Just state the result, no proof is necessary.)

6. You are playing a game in which you choose one of three doors and receive whatever sits behind the door. A $10,000 prize was randomly placed behind one of the doors; behind the other two doors are Bambi and Babbit, donkeys that, while adorable, are worthless to you.

After you pick your door, but before you open it, Monty Hall, your host who knows behind which door the prize lies, will open one of the doors you did not pick and with Bambi or Babbit behind it. At that point, if you prefer, you may change your pick of door to the one that remains.

Now, after you make your first pick of doors, but before Monty opens one of the donkey doors, Monty offers you a deal: Instead of opening one of the donkey doors and you possibly changing your pick of doors, he will pay you \( X \). You still get to keep your first pick of doors and still have a chance to win $10,000; you just don’t get to see one of the adorable donkeys before you pick.

(a) What is the minimum \( X \) required for you to accept Monty’s deal if you are risk neutral?

(b) What is the minimum \( X \) required for you to accept Monty’s deal if you have a utility function \( u = w^{1/2} \), and an initial wealth of $0?
Instructions. You have four hours to complete all the following questions. Point allocations are denoted in the questions. Please allocate your time wisely.

1. (20 points) Suppose the economy starts in steady state in which the government spends nothing and imposes no taxes. In this economy, identical households provide firms with a fixed amount of labor $L$ and buy consumption $C$ and investment goods $I$. Identical firms produce output $Y = F(K, L)$, which is divided among consumption, investment, and government goods. The production function $F$ is a standard concave constant-returns-to-scale function. Firms rent capital and labor from the households to carry out production. The households own the firms. There is no population growth. Capital depreciates at the constant proportional rate $0 < \delta < 1$ and so changes as $\frac{dK}{dt} = I - \delta K$. All markets are competitive.

Unexpectedly, the government decides it must provide national defense. It decides to do that by imposing a strictly proportional income tax with tax rate $\tau$ and spend whatever revenue it receives on national defense. Suppose the increase in taxes (from zero to a positive value) is permanent and is always expected to be so. National defense has no direct effects on either utility or productive efficiency and so does not enter the utility function or the production function.

(a) (8 points) Use the central planner version of the Cass model to derive the market equilibrium solutions for the complete time paths of the capital/labor ratio $k$, consumption per person $c$, and the fraction $g$ of output going to national defense.

(b) (4 points) Explain very briefly why the central planner approach is legitimate here as a method for obtaining the market equilibrium solution of the model.

(c) (8 points) Suppose that instead of fixing the tax rate and spending whatever revenue it gets, the government decides to fix defense expenditure at amount $G$ and adjust the tax rate $\tau$ as necessary to pay for it. Derive the complete time paths for $k$, $c$, and $g$. Explain those parts of the derivation that are different from part (a), and explain the economic reasons for any differences between the new time paths and the paths in the previous answer.

2. (20 points) Consider the following closed economy. Households are identical with intertemporal utility

$$U_t = \int_1^\infty \left[ \frac{C_{t+j}^{1-\theta} - 1}{1 - \theta} + M \frac{(1 - L_{t+j})^{1-\theta}}{1 - \theta} \right] e^{-\rho j} dj$$

and intertemporal flow budget constraint

$$\dot{B}_{t+j} = r_{t+j} B_{t+j} + w_{t+j} L_{t+j} - C_{t+j}$$

where $\theta$, $M$, and $\rho$ are constants. $C$ is consumption, $L \in [0, 1]$ is the fraction of time devoted labor, $B$ is bonds, $r$ is the interest rate, and $w$ is the wage rate. Firms have the following production technology:

$$Y_{t+j} = AL_{t+j}$$

where $Y$ is output and $A$ is a constant. The only use for output is consumption.

Solve for $C_{t+j}$, $L_{t+j}$, $B_{t+j}$, and $r_{t+j}$. Explain what you do to get the solutions, which means you must give the reason for each set of steps (e.g., "The first-order conditions are ...") or "We assume rational expectations. This implies that the household's expected value of stargazing is $X^e$ is the mathematical expectation of the model's solutions, $EX_t = a_0 + a_1 X_{t-1}$") and for any extra conditions you introduce (e.g., "I assume perfect competition among firms producing good $Y$ so that firms take prices as given.")

You should not explain what is trivially obvious (such as "Now we move the $X$ on the left side of the equation to the right side to get all the $X$ terms grouped together.")
3. (20 points) Consider the following stochastic growth model with inelastic labor supply. The economy is populated with many identical individuals who live forever. Households choose \( (c_t, k_t, b_t) \) to maximize

\[
E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}, \quad 0 < \beta < 1, \quad \gamma > 0
\]

subject to

\[ c_t + k_t + T_t + b_t = (1 - \tau_t)k_{t-1}^\alpha + r_{t-1}b_{t-1} \]

Capital \( k_t \) depreciates completely and a lump-sum tax \( T_t \) adjusts to balance the government’s budget constraint each period. \( \{\tau_t\} \) is a stochastic proportional tax rate, with \( \tau_t \in [0, \beta) \) for all \( t \). \( b_t \) is real government bonds issued in period \( t \) that pay a gross rate of return of \( \tau_t \) in period \( t + 1 \). \( k_0 > 0 \) is given and \( \alpha \in (0, 1] \).

(a) (3 points) Set up the Bellman equation for this problem.

(b) (2 points) Write down the government’s budget constraint.

(c) (5 points) Log-linearize the equilibrium conditions of the model and derive a second-order difference equation in log deviations of capital from its steady state.

(d) (4 points) Denote the roots of the equation you derived in part c by \( \lambda_1 \) and \( \lambda_2 \), and assume \( |\lambda_1| < 1/\beta \). Solve for the decision rule for capital. You do not need to derive explicit expressions for the \( \lambda \)'s to do this. Give an economic interpretation of your result.

(e) (4 points) Assume that \( \tau_t \equiv \ln \tau_t - \ln \bar{\tau} \) is an i.i.d. mean zero random variable where \( \bar{\tau} \) is the steady state value of \( \tau_t \). What is the effect of a sudden increase in \( \tau_t \) on capital accumulation? Explain the economic intuition for this result.

(f) (2 points) Instead of levying lump-sum taxes, the government would like to issue money to finance its budget. Under what conditions, if any, is this feasible? If it is infeasible, state a modification to the model that would make this possible.

4. (20 points) Assume an economy can be represented by the following model. Money demand is given by:

\[
m_t - p_t = y_t - \lambda t + v_t, \quad \lambda > 0,
\]

where \( m_t \) denotes nominal money balances, \( p_t \) is the price level, \( y_t \) is current output, \( i_t \) is the nominal risk-free interest rate, and \( v_t \) is an i.i.d. shock such that \( E_t v_{t+1} = 0 \). Let \( w_t \) be the nominal wage. It is set at time \( t - 1 \) to equal the price level expected for the following period:

\[
w_t = E_{t-1} p_t.
\]

Aggregate demand is given by

\[
y_t^d = \delta (E_t p_{t+1} - p_t) + g_t, \quad \delta > 0,
\]

where \( g_t \) is an i.i.d. demand shock such that \( E_t g_{t+1} = 0 \). The aggregate supply is given by

\[
y_t^s = \theta (p_t - w_t), \quad \theta > 0.
\]

The monetary authority sets the interest rate according to a Taylor-type rule:

\[
i_t = \psi p_t + u_t, \quad \psi > 0,
\]

where \( u_t \) is an i.i.d. monetary policy shock such that \( E_t u_{t+1} = 0 \). In equilibrium, \( y_t = y_t^d = y_t^s \). Assume all shocks are uncorrelated with each other and \( \theta > \delta (1 + \psi) \).

(a) (4 points) Give a theoretical rationale for equation (2).
(b) (8 points) Derive a solution for equilibrium output and the price level in terms of the exogenous variables and parameters of the model.

(c) (4 points) Do the 
shocks from the money demand equation affect equilibrium output? Explain the economic intuition for this result.

(d) (4 points) In the United States, inflation and output were more volatile over the period 1970-1982 than the period 1983-2008. Give an economic explanation consistent with this model that could reproduce this fact.

5. (20 points) In the Solow growth model, the evolution of the capital/labor ratio is given by

$$\dot{k} = sf(k) - (\delta + n)k$$

where $s$ is the constant, exogenous saving rate, $f$ is a standard concave homogeneous degree 1 (i.e., constant returns to scale) production function, $\delta$ is the depreciation rate of capital, and $n$ is the population growth rate. In the Cass growth model, the evolution of capital is given by

$$\dot{k} = f(k) - c - (\delta + n)k$$

where that path of consumption $c$ is chosen to maximize lifetime utility

$$U = \int_0^\infty u(c_t) e^{-\rho t} dt$$

where the instantaneous utility function $u$ is concave and the rate of time preference $\rho$ is positive.

(a) (8 points) In the Solow model, solve for the Golden Rule level of $k^*_\text{Solow}$, that is, the steady state value of $k$ at which consumption $c$ is maximized. Because you have not been given the specific production function, you cannot actually find $k^*_\text{Solow}$, but you can find an expression for it that you could solve if you were given a specific production function. Thus what you need to do is find that expression. [Hint: It will be an expression for the marginal product of capital evaluated at $k^*_\text{Solow}$.]

(b) (8 points) In the Cass model, solve for the steady state value of $k^*_\text{Cass}$. Again, you can’t actually find $k^*_\text{Cass}$ because you don’t have a specific form for $f$, but you can find an expression for $f'(k^*_\text{Cass})$ that implies the value for $k^*_\text{Cass}$.

(c) (4 points) Explain the economic intuition behind the relation between $k^*_\text{Solow}$ and $k^*_\text{Cass}$. 
1. A consumer allocates all his income, \( m \), to the consumption of two goods, \( x_1 \) and \( x_2 \). The market prices of the two goods are \( p_1 \) and \( p_2 \). The consumer’s utility from the consumption of the two goods is given by:

\[
U(q_1, q_2) = q_1^a q_2^{(1-a)}.
\]

Suppose that the price of the first good rises from \( p_1^* \) to \( p_1^1 \). The price of the second good doesn’t change.

(a) Derive an exact expression for the change in consumer surplus induced by the price change. Derive an exact expression for the compensating variation of the price change.

(b) Which of the two measures derived in part (a) is larger in magnitude? Why?

(c) Suppose that the consumer’s income is $100, the share of the consumer’s budget spent on the first good is \( \frac{1}{2} \), and the increase from \( p_1^* \) to \( p_1^1 \) represents a 10% change. Without using a calculator, approximate to the nearest whole dollar the change in consumer surplus.

2. Suppose a parent and a child play the following game. First, the child takes an action \( A \) that produces income for the child, \( I_C(A) \), and income for the parent, \( I_P(A) \). (Think of \( I_C(A) \) as the child’s income net of any costs of the action \( A \).) Second, the parent observes the incomes \( I_C \) and \( I_P \) and then chooses a bequest, \( B \), to leave to the child. The child’s payoff is \( U(I_C + B) \); the parent’s is \( \min \{ I_P - B, I_C + B \} \). Assume that: the action is a nonnegative number, \( A \geq 0 \); the income functions \( I_C(A) \) and \( I_P(A) \) are strictly concave and maximized at \( A_C > 0 \) and \( A_P > 0 \), respectively; the bequest \( B \) can be positive or negative; and the utility function \( U \) is increasing and strictly concave.

(a) Prove the “Rotten Kid” Theorem: in the backwards induction outcome, the child chooses the action that maximizes the family’s aggregate income, \( I_C(A) + I_P(A) \), even though only the parent’s payoff exhibits altruism.

(b) What are the important assumptions this model makes that might change your answer to (a) if they were relaxed?
3. Two hundred entrepreneurs have access to a technology that produces widgets. The costs of using the technology are given by:

\[ C = y^2, \]

where \( y \) is the quantity of widgets produced.

One hundred of the entrepreneurs can either produce widgets according to the technology given above, or they have alternative employment that would pay them $4 working as gardeners. The other one hundred entrepreneurs are brain surgeons, who would earn $9 if they weren’t engaged in the manufacture and sale of widgets.

The market demand for widgets is given by:

\[ Q = 1600 - 100P, \]

where \( Q \) is the total quantity demanded and \( P \) is price.

(a) Derive and illustrate the market supply curve for widgets.

(b) What are the market equilibrium price and quantity? How many of the gardeners choose to produce widgets and how much do each of them produce? How many of the brain surgeons choose to produce widgets and how much do each of them produce?

(c) Now suppose that buyers of widgets are taxed at the rate of $6 per widget. What are the market equilibrium price and quantity? How many of the gardeners choose to produce widgets and how much do each of them produce? How many of the brain surgeons choose to produce widgets and how much do each of them produce?

(d) In the after-tax situation, how much rent from widget production do each of the entrepreneur/surgeons earn?

4. Consider the following exchange economy. There are 2 consumers with identical preferences over two consumption goods, \( x_1 \) and \( x_2 \), given by:

\[ u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2} \quad (1) \]

Consumer 1 has endowment \( w_1 = (1, 0) \) and consumer 2 has endowment \( w_2 = (0, 1) \). (For reference, \( \sqrt{2} \approx 1.41, \sqrt{3} \approx 1.73, \sqrt{5} \approx 2.24, \sqrt{7} \approx 2.65 \))

(a) Calculate the set of Pareto efficient allocations.

(b) Show that the allocation that gives \( (\frac{1}{4}, \frac{1}{4}) \) to consumer 1 and \( (\frac{3}{4}, \frac{3}{4}) \) to consumer 2 is in the core.

(c) Now consider the 2-replica economy where all four consumers have the same utility given by (1). Consumers 1 and 3 have the bundle \( (1, 0) \), and consumers 2 and 4 have the bundle \( (0, 1) \). Show that the allocation in part (b) is no longer in the core.
5. A monopolist serves two markets, Fresno and Bakersfield. Demand curves in the two markets are given by:

Fresno demand: \( Q_F = 20 - P_F \)

Bakersfield: \( Q_B = 10 - P_B \).

The marginal cost of production is constant at 4 per unit. There are no fixed costs.

(a) If the monopolist can charge different prices in the two markets, what are the prices it should charge?

(b) If the monopolist is unable to price discriminate, what is the single best price it should charge?

Now suppose that production is costless: fixed and marginal costs are zero.

(c) If the monopolist can charge different prices in the two markets, what are the prices it should charge?

(d) If the monopolist is unable to price discriminate, what is the single best price it should charge?

(e) In the zero-cost situation, which is the more efficient outcome—the price discriminating outcome in (c) or the single-price monopoly outcome in (d)? Which of (c) and (d) yield the higher profits for the monopolist?

6. Two buyers each obtain a private signal about the market value of an object. The signals are independent and can be either high (\( H \)) with probability .5 or low (\( L \)) with probability .5. If both obtain signal \( H \), the market value of the object is 1. If either obtains signal \( L \), then the market value is 0 (for example, if buyer 1 receives signal H and buyer 2 receives signal L, then the market value of the object is 0).

(a) What is the expected value of the object to a buyer who sees signal \( L \)? To a buyer who sees signal \( H \)?

(b) Suppose we auction the item using a sealed-bid, first-price auction and suppose buyers bid their expected value computed in part (a). What are their expected profits if they receive signal \( H \)?

(c) Suppose we auction the item using a sealed-bid, second-price auction. What are the equilibrium bidding functions?
Instructions. Do all questions. Point values are shown in parentheses. You have 4 hours to complete the exam. If you know part but not all of the answer, put down what you know. A blank answer is guaranteed to earn no points, so even if you don’t know the complete answer, be sure to tell us what you do know. However, do not guess if you don’t know. Irrelevant statements will not hurt your score, but wrong statements will cost you points and so will be worse than saying nothing.

1. (24 points) The macroeconomic effects of alternative tax policies.

(a) Assume the following simple economy:

1. Competitive markets for all goods.
2. A fixed number of identical households with the standard concave intertemporal utility function
   \[ U = \int_0^\infty u(C_t) e^{-\rho t} dt \]
   where the intra-temporal utility function \( u(C) \) is globally increasing and concave in consumption per person \( C \).
3. Firms supply output according to the strictly concave, constant-returns-to-scale production function
   \[ Y_t = F(K_t, L_t) \]
   where \( K \) is physical capital and \( L \) is labor and where \( F \) is globally increasing in both \( K \) and \( L \).
4. Both \( u(C) \) and \( F(K, L) \) satisfy the Inada conditions.
5. Output can be used for consumption or investment.
6. Capital evolves according to the usual accumulation equation
   \[ \dot{K}_t = I_t - \delta K_t \]
   where \( \delta \) is the rate of depreciation. Investment is irreversible (i.e., \( K \) cannot be converted to \( C \), which means \( I_t \geq 0 \).
7. Households own the firms and thus the capital, which they rent to firms at the going rental rate. Households also supply a fixed amount of labor \( L_t = L \) for which they are paid the going wage.
8. Government collects taxes and spends the revenue on a fixed amount \( G \) of national defense. There is no government debt or money, so the government’s budget always is balanced. National defense is not a factor of production and not an argument in the utility function. (Actually, because it is constant in this problem, it wouldn’t matter if \( G \) were an argument of either the production function or the utility function.) Taxes take one of two forms:
   1. A consumption tax:
      \[ T_t = \tau_C C_t \]
      where \( T \) is tax revenue, \( \tau_C \) is the consumption tax rate (the same for all individuals and constant over time)
   2. An income tax:
      \[ T_t = \tau_Y Y_t \]
      where \( \tau_Y \) is the same for all individuals and constant over time.

(b) Use the continuous-time Cass model to address the following issues concerning this economy’s steady state behavior.

1. (10 points) Show how the levels of output and capital differ under the two tax policies.
2. (6 points) What is the economic intuition for the differences under the two policies?
3. (4 points) What is the implicit solution for $\tau_C$? You cannot give a closed-form solution because you have not been given the specific utility and production functions. Instead, derive an expression for $\tau_C$ that depends only on parameters and exogenous variables.
4. (4 points) What is the implicit solution for $\tau_Y$?

2. (20 points) Conditions for a competitive equilibrium and its properties.
   
   (a) Consider the same economy as in the previous question with the following modifications:
   1. There is no government.
   2. Labor is variable and chosen by the households, so the intratemporal utility function is the concave function $u(C, L)$, increasing in $C$ and decreasing in $L$.
   
   (b) Answer the following questions about this economy.
   1. (4 points) Is this economy guaranteed to have a competitive equilibrium? [Note: The question is about competitive equilibrium, not the steady state of the dynamic system. That question comes later.] Why or why not?
   2. (4 points) If there is a competitive equilibrium, is it unique? Why or why not?
   3. (3 points) What can you say about the stability of any competitive equilibria that exist?
   4. (3 points) When the conditions for existence, uniqueness, and stability are satisfied, does this economy have a steady state? Why or why not?
   5. (3 points) If there is a steady state, is it unique? Why or why not?
   6. (3 points) Explain whether each steady state that exists is dynamically stable or unstable.

3. (21 points) Assume that there are a continuum of identical, infinitely lived households on the unit interval. Household $i$ maximizes

$$E_0 \sum_{i=0}^{\infty} \beta^i \frac{1}{1-\gamma} \left( C_i^t - \psi \frac{(L_i^t)^{1+\kappa}}{1+\kappa} \right)^{1-\gamma}$$

where $C_i^t$ is consumption and $L_i^t$ is labor. The household maximizes lifetime utility subject to the real budget constraint

$$C_i^t + B_i^t + K_{t+1}^i = W_i L_i^t + R_i K_i^t + R_{t-1} B_{t-1}^i$$

where $B_i$ are real private bonds purchased at time $t$ that give a risk-free rate of return $R_t$ in period $t+1$, $K_i^t$ represents the capital stock the household has accumulated and rents to firms with gross real return $R_i K_i^t$, and $W_i$ is the real wage.

A perfectly competitive representative firm produces output according to the technology $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$ where $\ln A_t$ is a mean zero, i.i.d technology productivity shock, $K_t \equiv \int_0^1 K_i^t dt$, and $L_t \equiv \int_0^1 L_i^t dt$. Assume that $\psi, \kappa, \gamma > 0$ and $\beta, \alpha \in (0, 1)$.

(a) (6 points) Solve for the $i$th household’s and representative firm’s first order conditions.
(b) (4 points) List all market clearing conditions.
(c) (3 points) What is the relationship between the expected real return on bonds and capital? Give an economic explanation for this relationship.
(d) (3 points) Solve for the steady state values of $\frac{K}{Y}$, $\frac{L}{Y}$, and $\frac{C}{Y}$.
(e) (3 points) Derive the log-linearization policy functions for $\hat{L}_t$ and $\hat{Y}_t$ ("hats" here denote log deviations from steady state).
(f) (2 points) Suppose that household consumption utility and labor disutility depend on the previous aggregate levels of consumption and labor in the economy, \( C_{t-1} \) and \( L_{t-1} \), which each individual household takes as given. Lifetime utility is now given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left( C_t^\gamma - C_{t-1} - \psi \frac{(L_t^\gamma - L_{t-1})^{1+\kappa}}{1+\kappa} \right)^{1-\gamma}.
\]

Will this modification change the log-linearized policy function for \( \dot{L}_t \) that you derived in part (e)? Explain why or why not.

4. (15 points) Uncertainty and consumption.

The consumption Euler equation is

\[
u'(c_t) = E u'(c_{t+1}) \prod_{j=0}^{j-1} \frac{(1 + r_{t+j})}{(1 + \rho)^{j+1}} \]

where \( \rho \) is the rate of time preference and \( r \) is the interest rate. Suppose there is an increase in the uncertainty about the future course of the economy and thus an increase in the uncertainty about future values of consumption.

(a) (6 points) Under what conditions will the slope of the time path of the household’s planned consumption increase, decrease, or remain the same?

(b) (3 points) What is the economic intuition for the foregoing result?

(c) (6 points) Explain what would happen to the interest rate in each of the three cases.

5. (20 points) Consider an economy with a continuum of households indexed by \( j \in [0, 1] \), where each household differs only in that it supplies differentiated labor services. Household \( j \)'s utility is given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t(j)^{1-\gamma} - 1}{1-\gamma} - \theta \frac{n_t(j)^{1+\kappa}}{1+\kappa} \right]
\]

where \( C_t \) is real consumption and \( n_t \) is real labor services provided. Household \( j \)'s real budget constraint in period \( t \) is given by

\[
C_t(j) + \frac{B_t(j)}{P_t} = \frac{R_{t-1} B_{t-1}(j)}{\pi_t} + \frac{u_t(j) n_t(j)}{P_t} + d_t(j)
\]

where \( B_t \) denotes nominal private bonds with a risk-free gross nominal return of \( R_t \) in period \( t+1 \), \( P_t \) is the price of goods, \( \pi_t \) is the inflation rate, \( u_t \) is the nominal wage for household \( j \)'s labor, and \( d_t \) denotes real dividends received from firm profits. Note that since the household provides a differentiated labor service, it has monopoly power in its supply of labor and sets its nominal wage. Assume that nominal wages adjust according to a Calvo pricing mechanism. Assume that households set their wage taking into account the demand for labor services, given by

\[
n_t(j) = N_t \left( \frac{u_t(j)}{W_t} \right)^{-\sigma}
\]

where \( N_t \) is aggregate labor services provided by all households and \( W_t \) is the aggregate nominal wage rate. The details of the set-up of the firms and monetary authority is inconsequential for the questions below. For simplicity, assume that goods prices are fully flexible and that the only source of uncertainty is a technological productivity shock.

(a) (4 points) In general, households do not like volatility in this model set-up. Will inflation volatility be more, less, or equally welfare reducing for an individual household in this sticky wage economy than an economy with flexible wages? Explain your answer. You do not need to discuss aggregate welfare in the economy in your answer.
(b) (5 points) Can the equilibrium dynamics be derived from the decisions of a representative household? Explain why or why not.

(c) (5 points) Provide additional assumptions that will ensure that consumption growth of households \( i \) and \( j \) are equal. Show that under your assumptions

\[
\frac{C_{t+1}(i)}{C_t(i)} = \frac{C_{t+1}(j)}{C_t(j)}.
\]

(d) (6 points) Show that the expectations hypothesis of the term structure of interest rates holds in this model by introducing a 2-period nominal bond \( B_t^2 \) that pays a gross risk-free rate of return of \( R_t^{2*} \) in period \( t + 2 \).
Instructions. Do all questions. Point values are shown in parentheses. You have 4 hours to complete the exam. If you know part but not all of the answer, put down what you know. A blank answer is guaranteed to earn no points, so even if you don’t know the complete answer, be sure to tell us what you do know. However, do not guess if you don’t know. Irrelevant statements will not hurt your score, but wrong statements will cost you points and so will be worse than saying nothing.


Consider a perfect foresight, deterministic economy populated by a continuum of identical households on the unit interval. Utility is given by

\[ \sum_{t=0}^{\infty} \beta^t [u(C_t) - v(H_t)] \]

where \( C_t \) denotes consumption and \( H_t \) denotes hours worked. Assume \( u'(C) > 0, u''(C) < 0, v'(H) > 0 \), and \( v''(H) < 0 \). Production of output \( Y_t \) occurs via the simple production function \( f(H_t) \). Let \( P_t \) denote the price of output. In addition to private consumption, the government has public expenditure \( G_t \). There is no capital stock or investment in the economy. The government issues one period nominal government bonds \( B_t \) that pay a gross nominal interest rate of \( i_t \) in period \( t + 1 \). In addition, the government collects nominal lump-sum taxes \( P_t T_t \) from the households.

(a) (4 points) Write down the aggregate resource constraint for the economy.

(b) (4 points) Assume prices are fully flexible. Solve for the equilibrium conditions of the model and use them to characterize the effect of an exogenous, one-time, permanent change in government expenditures on output. Assume that the lump-sum tax finances the change in government spending. That is, solve for \( dY_t/dG_t \) and determine if it is positive, negative, less than one in absolute value, or greater than one in absolute value. Give an economic explanation for the result.

(c) (4 points) Assume that there are a continuum of firms on the unit interval producing differentiated goods with the same technology. That is, the \( i \)th firm produces \( Y_i(t) \) via the production function \( f(H_i(t)) \). Note that since each firm provides a differentiated good, it has monopoly power in its supply of the good and sets its good’s price. Assume that prices are still fully flexible each period and that the labor market is perfectly competitive. Assume that households derive utility from all goods and let \( C_t \) now represent a CES aggregation of these individual goods. Will your result from part (b) remain the same or not? Give an economic explanation for your reasoning.

(d) (4 points) Assume that a monetary authority implements a policy to keep the gross real interest rate in the economy constant at all times at \( \bar{R} = \frac{3}{10} \). Notice that we are not providing the micro foundation for this assumption, but simply taking as given that it is somehow possible. Will your result from part (b) remain the same or not? Give an economic explanation for your reasoning.

(e) (4 points) Continue to assume the monetary policy of part (d). Show that under certain parameter restrictions consumption follows a random walk. Be sure to state the parameter restrictions you need for the result to hold.

2. Household choice and macroeconomic behavior.

Suppose households are identical. They supply 1 unit of labor at all times and receive the market wage \( w \) in return. They borrow and lend among each other at the market interest rate \( r \). The value of their assets at any moment is \( A \), which may be positive or negative. They choose consumption paths to maximize lifetime utility

\[ U = \int_{t_0}^{\infty} u(c_t) e^{-\rho t} \, dt \]

subject to their lifetime budget constraints, where \( u \) is a standard concave utility function, \( c \) is consumption, and \( \rho \) is the rate of time preference.
(a) (5 points) Write the full continuous time optimal control (i.e., maximum principle) statement of the household’s choice problem, stating the Hamiltonian, dynamic equations, first-order conditions, and endpoint conditions.

(b) (5 points) Assume the economy starts in steady state with $r = \rho$. Prove formally that the optimal solution for the household’s lifetime plan is to have constant consumption at all times.

(c) (5 points) Suppose one and only one household receives a big influx of income in the initial period and only in the initial period from a child living and working in another country. Use the model you have developed to this point to show what change in the household’s consumption path an outside observer would see.

(d) (5 points) Suppose now that all households receive the same influx of income from abroad, only in the initial period, from their children working in other countries. Explain what change in the household’s consumption path an outside observer would see in this case. Explain the economics behind the similarities and differences between this and the previous case.

3. Countercyclical fiscal policy.

The US economy was in recession during 2008 and has been sluggish since then. In response to the recession, the Obama administration urged the US Congress to enact a stimulus program, which the Congress did. The stimulus program comprised a mix of federal transfers and purchases, together with a tax policy. Consider the following cases that look at different parts of the stimulus program.

(a) (5 points) Start by analyzing the effect of the recession itself. The recession was caused largely by turmoil in the financial industry. Suppose we can treat that turmoil as a temporary negative productivity shock. Assume the aggregate production function takes the form $Y = AF(K)$. Use the Cass model to show the effect of a temporary reduction in the TFP parameter $A$ on the economy’s time path.

(b) (5 points) Most of the spending increase in the federal stimulus program took the form of transfers to individuals. For now assume that increase was funded by a simultaneous increase in personal income taxes. Use the Cass model to explain the effect of those parts of the stimulus program on the time path of the economy. For this part, assume there was no recession, just a change in federal fiscal policy.

(c) (5 points) In reality, there was no concurrent increase in taxes but rather an increase in the amount of federal debt outstanding. Use the Cass model to show the effect of the stimulus program on the time path of the economy in that case. Remember to take into account the government’s flow budget constraint and its lifetime budget constraint. Again assume for the time being that there was no recession, just the change in fiscal policy.

(d) (5 points) Now put together your answers to parts (a) and (c) to explain the expected time path of the economy.

[Here’s something to do on your own after the exam if you like using economic theory to analyze reality. Extend the analysis a bit by considering that a small part of the stimulus spending was for federal purchases, few of which were for goods that directly affect productivity. Add that to the story for a fairly complete picture of the expected effects of the stimulus program.]

4. Monetary policy.

Consider a simple New Keynesian model. The log-linearized Phillips curve in this economy is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(x_t - e_t),$$  

(1)
where $x_t$ is total output, $\pi_t$ is inflation for the aggregate price index, and $e_t$ denotes an i.i.d. mean zero supply shock that hits all firms symmetrically. In addition, the economy is characterized by the goods’ demand-side (IS) equation:

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}),$$

(2)

where $i_t$ is the gross nominal interest rate; the money demand equation:

$$m_t = \gamma x_t - \frac{\beta}{(1 - \beta)} i_t$$

(3)

$m_t$ is real money balances; and the monetary policy:

$$i_t = \phi_x \pi_t$$

(4)

All variables are expressed as log deviations from the deterministic steady state.

(a) (6 points) Give an economic explanation as to why $\phi_x > 1$ is necessary to ensure that the equilibrium will be unique. You do not have to derive this condition mathematically, but rather explain the intuition for its necessity.

(b) (7 points) Suppose that the monetary authority cares about inflation variability, output variability, and interest rate variability. Instead of following the simple rule given by equation (2), the objective of the monetary authority is to minimize

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{1}{2} [\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2].$$

Derive the optimal monetary policy assuming that the monetary authority operates under discretion. Give an economic explanation for the optimal monetary policy rule.

(c) (7 points) Suppose that instead of following the rule given by equation (2), the monetary authority follows the rule:

$$\mu_t = \rho \mu_{t-1} + \nu_t, \quad 0 < \rho < 1,$$

where $\mu_t$ is the rate of nominal money growth and $\nu_t$ is an i.i.d mean zero shock. Do you expect this model to generate a negative correlation between money growth and nominal interest rates? Would you expect the answer to change if prices were fully flexible? Explain your reasoning.

5. Endogenous growth.

Suppose the aggregate production function is

$$Y = AK$$

where $Y$ is aggregate output, $A$ is a constant, and $K$ is the aggregate capital stock. The representative household chooses a path of consumption to maximize lifetime utility

$$U = \int_{t_0}^{\infty} u(C_t) e^{-\rho t} dt$$

subject to the flow budget constraint

$$\dot{K} = Y - C - \delta K$$

where $u$ is constant relative risk aversion (CRRA) utility function, $C$ is consumption, $\rho$ is the rate of time preference.

(a) (6 points) A balanced growth path has the property that the growth rates of all variables are constant over time (and may be zero for some of the variables). Prove that in this economy $Y$, $C$, and $K$ all grow at the same rate along the balanced growth path.
(b) (6 points) Derive the growth rate of $Y$. [Hint: Even if you can’t do part (a), you can use the result from it here.]

(c) (8 points) Explain the economic reason this economy has perpetual growth even though, in sharp contrast to the Cass and Solow models, there is no exogenous growth in $A$. [That, by the way, is why the Solow and Cass “growth” models are not models of growth at all. They are merely models with growth that is exogenous and unexplained. In contrast, growth emerges endogenously from the $AK$ model used here and from other more realistic models of the endogenous growth class.]
1. Kim receives a fixed pre-tax income equal to \( I \) and pays taxes in proportion to her taxable income at the rate of \( r \) per dollar. She allocates her after-tax income to expenditures on a market good that she purchases at a price of \( p \) per unit and to donations to her favorite charity, the Red Cross. Contributions to the Red Cross are tax deductible: she is only taxed on her income net of her contributions.

Kim’s preferences over \( C \) and \( M \) are given by:

\[
U(C, M) = C^\alpha M^\beta, \quad \alpha + \beta = 1,
\]

where \( C \) denotes dollar contributions to the Red Cross and \( M \) denotes the quantity purchased of the market good.

(a) How does a change in \( r \) affect the relative price to Kim of \( C \) and \( M \)?

(b) What are the effects of an increase in the tax rate, \( r \), on Kim’s charitable contributions? What are the effects on her consumption of the market good?

2. A buyer and a seller have valuations \( v_b \) and \( v_s \). It is common knowledge that there are gains from trade (i.e. that \( v_b > v_s \)), but the size of gains is private information as follows: the seller’s valuation is uniformly distributed on \([0, 1]\); the buyer’s valuation \( v_b = k \cdot v_s \); \( k > 1 \) and both the buyer and seller know what \( k \) is; the seller knows \( v_s \) (and hence \( v_b \)) but the buyer does not know \( v_s \) (or \( v_b \)). Suppose the buyer makes a single offer \( p \), which the seller either accepts or rejects. What offer \( p \) does the buyer make in equilibrium?

3. Menhaden is a species of fish caught in the Atlantic Ocean. When menhaden are caught, they are processed by a competitive industry into fish meal—an ingredient in animal feed—and fish oil—consumed by humans as a vitamin supplement. One pound of Menhaden fish (\( F \)) yields 3/4 of a pound of meal (\( M \)) and 1/4 of a pound of oil (\( O \)) in fixed proportions.

(a) Suppose that new research demonstrates that fish oil vitamin supplements are more beneficial to human health than was previously thought. Analyze how this affects the economic welfare of menhaden producers (fishermen). Analyze how this affects the economic welfare of fish meal consumers?

(b) Suppose that the government imposes a per-unit tax on fish meal sales. Analyze the welfare effects of the tax on: menhaden producers, fish meal purchasers, and fish oil consumers.
4. Initech has a world monopoly on the personal computer market. It can make two types of
computers: high-end and low-end. Buyers are either casual (C) users or intensive (I) users. There
is a total population of \( P \) and \( c \) percent of the people are casual users. The cost of producing
computers as well as the benefit to the different types of consumers are given below:

<table>
<thead>
<tr>
<th></th>
<th>Cost</th>
<th>Benefit to C-type</th>
<th>Benefit to I-type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-end Computers</td>
<td>1</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>High-end Computers</td>
<td>3</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

Each consumer chooses the machine that maximizes benefit minus price.

(a) If Initech knows the type of each consumer, what machine does it sell to each type of
consumer and at what price?

(b) Suppose that Initech cannot tell which type a consumer is. Instead it offers a menu of
prices. For each \( c \), what is the optimal menu of prices?

5. All producing and potentially producing firms in an industry have access to the same technology,
given by:

\[ y = x^\alpha, \]

where \( y \) is the quantity of output produced per period by a firm and \( x \) is the quantity of the single
input employed per period. The parameter \( \alpha \) lies in the interval \((0,1)\).

Firms purchase the input at price \( w \), which is not affected by the quantity purchased by the
industry. In addition to the costs of purchasing \( x \), a firm that decides to produce incurs a cost
equal to \( f \), which is independent of the quantity produced. Firms sell their output at price \( p \),
which they take as fixed.

(a) Derive an expression for long-run equilibrium output price, \( p^* \). Derive the comparative
static effect on \( p^* \) of a change in \( f \). Derive the comparative static effect on \( p^* \) of a change
in \( w \).

(b) Suppose that demand for the industry’s output is given by:

\[ Y = D(p) = 1/P^\delta, \]

where \( \delta > 0 \).

Derive an expression for the long-run equilibrium number of firms in the industry, \( n^* \).
Derive the comparative static effect on \( n^* \) of a change in \( f \). Derive the comparative static
effect on \( n^* \) of a change in \( w \).
6. Consider a first-price auction with three bidders, in which the bidders' types are i.i.d. random variables uniformly distributed on the interval [0,1]. Each bidder knows her own type but only the distribution of her opponent's type, and each bidder is risk-neutral. The bidders simultaneously and independently submit sealed bids, and the high bidder wins the item and pays the amount of her bid. When bidder i's type equals $t_i$, her valuation from winning the item is given by $(t_i)^4 + \alpha$. Determine the Bayesian-Nash equilibrium of the first-price auction.